Go to
join.iClicker.com
HXFY

COS 484: Natural Language Processing

L2: n-gram Language Models

Spring 2025

Announcements / set up

- Join iClicker! We’re going to have some in-class polls today
- HW0 due next Monday, 1:59pm
- Office hours + locations are now available on the website

Go to

join.iClicker.com

HXFY

Lecture plan

- What is an n-gram language model?
- Generating from a language model
- Evaluating a language model (perplexity)
- Smoothing: additive, interpolation, discounting

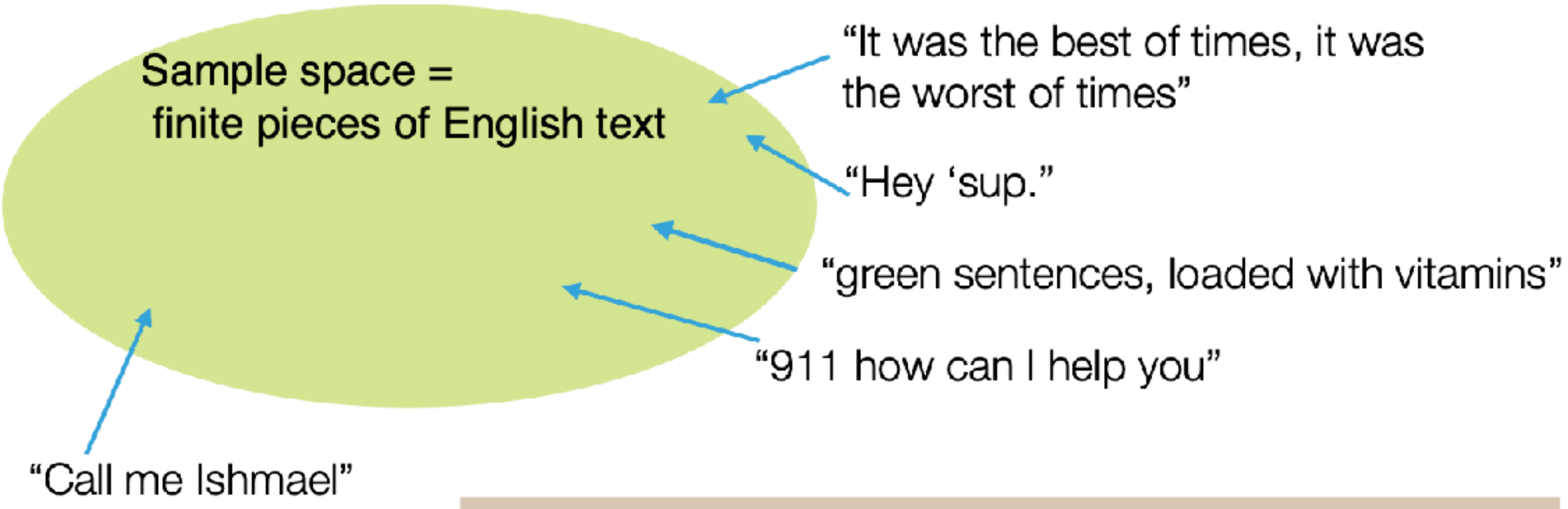
Recommended reading:
JM3 3.1-3.5

CHAPTER
3
N-gram Language Models

What is a language model?

- A probabilistic model of a sequence of words
- Joint probability distribution of words $w_1, w_2, \ldots, w_n$:

$$P(w_1, w_2, w_3, \ldots, w_n)$$



(i.e., $\mathrm{Pr}[w_1 w_2 w_2 \ldots w_n]$ associated with every finite word sequence $w_1 w_2 w_2 \ldots w_n$ (including nonsensical ones)

How likely is a given phrase, sentence, paragraph or even a document?

Chain rule

Conditional probability:
$p(w \mid w_1, w_2), \forall w \in V$

$$p(w_1, w_2, w_3, \ldots, w_n) =$$

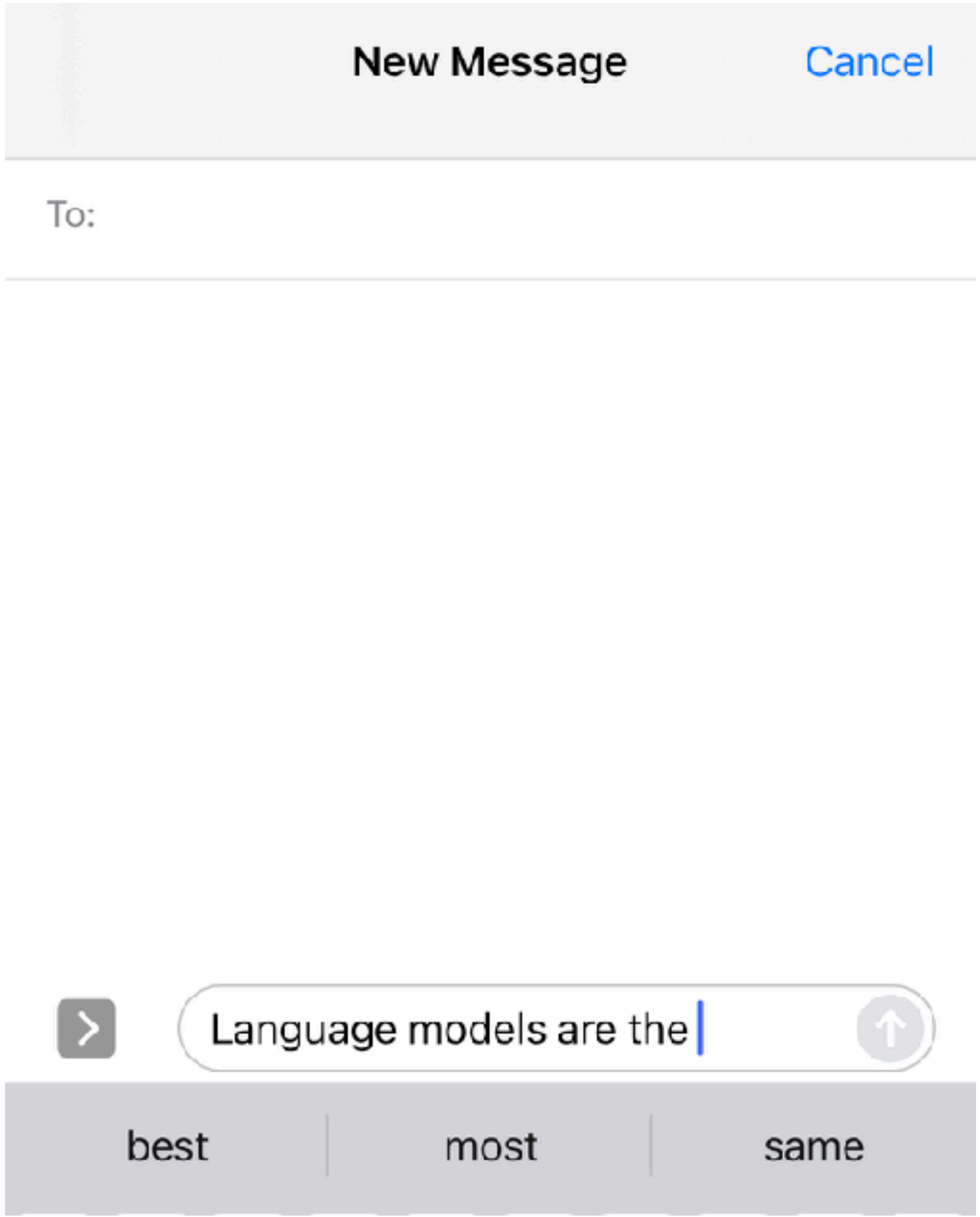
$$p(w_1)p(w_2 \mid w_1)p(w_3 \mid w_1, w_2) \times \cdots \times p(w_n \mid w_1, w_2, \ldots, w_{n-1})$$

Sentence: “the cat sat on the mat”

$$P(\text{the cat sat on the mat}) = P(\text{the}) * P(\text{cat}|\text{the}) * P(\text{sat}|\text{the cat})$$
$$*P(\text{on}|\text{the cat sat}) * P(\text{the}|\text{the cat sat on})$$
$$*P(\text{mat}|\text{the cat sat on the})$$

Implicit order

Language models are everywhere



Estimating probabilities

$$P(\text{sat}|\text{the cat}) = \frac{\text{count}(\text{the cat sat})}{\text{count}(\text{the cat})}$$

$$P(\text{on}|\text{the cat sat}) = \frac{\text{count}(\text{the cat sat on})}{\text{count}(\text{the cat sat})}$$

$$\vdots$$

Maximum likelihood estimate (MLE)

Assume we have a vocabulary of size V,

how many sequences of length n do we have?

A) $n * V$

B) n^V

C) V^n

D) V/n

Estimating probabilities

$$P(\text{sat}|\text{the cat}) = \frac{\text{count}(\text{the cat sat})}{\text{count}(\text{the cat})}$$

$$P(\text{on}|\text{the cat sat}) = \frac{\text{count}(\text{the cat sat on})}{\text{count}(\text{the cat sat})}$$

$$\vdots$$

Maximum likelihood estimate (MLE)

- With a vocabulary of size V, # sequences of length n = V^n
- Typical English vocabulary ~ 40k words
    - Even sentences of length ≤ 11 results in more than 4×10^{50} sequences. Too many to count!
    - *(For reference, # of atoms in the earth* $\sim 10^{50}$)

Markov assumption

- Use only the recent past to predict the next word

- Reduces the number of estimated parameters in exchange for modeling capacity

- 1st order

$$P(\text{mat}|\text{the cat sat on the}) \approx P(\text{mat}|\text{the})$$

- 2nd order

$$P(\text{mat}|\text{the cat sat on the}) \approx P(\text{mat}|\text{on the})$$

Andrey Markov

k^{th} order Markov

Consider only the last *k* words (or less) for context which implies the probability of a sequence is:

$$P(w_i \mid \boxed{w_1 w_2 \ldots w_{i-1}}) \approx P(w_i \mid \boxed{w_{i-k} \ldots w_{i-1}})$$

$$P(w_1 w_2 \ldots w_n) \approx \prod_i P(w_i \mid w_{i-k} \ldots w_{i-1})$$

(assume $w_j = \phi \quad \forall j < 0$)

Need to estimate counts for up to (k+1) grams

n-gram models

Unigram $$P(w_1, w_2, ... w_n) = \prod_{i=1}^{n} P(w_i)$$ e.g. P(the) P(cat) P(sat)

Bigram $$P(w_1, w_2, ... w_n) = \prod_{i=1}^{n} P(w_i | w_{i-1})$$ e.g. P(the) P(cat | the) P(sat | cat)

and Trigram, 4-gram, and so on.

Larger the n, more accurate and better the language model
(but also higher costs)

Caveat: Assuming infinite data!

Estimating probabilities

Consider the following corpus

<s> I like apples </s>

<s> You like strawberries </s>

<s> You like apples </s>

Note: <s> and </s> are starting and ending tokens

What's the bigram probability P(apples | like) ?

(A) 1/3 (B) 2/3 (C) 1/2 (D) 1

Estimating probabilities

Consider the following corpus

<s> I like apples </s>

<s> You like strawberries </s>

<s> You like apples </s>

Note: <s> and </s> are starting and ending tokens

What's the bigram probability P(apples | like) ?

(A) 1/3 (B) 2/3 (C) 1/2 (D) 1

$$P(\text{apples} \mid \text{like}) = \frac{\text{Count("like apples")}}{\text{Count("like")}} = \frac{2}{3}$$

Estimating probabilities

Consider the following corpus

<s> I like apples </s>

<s> You like strawberries </s>

<s> You like apples </s>

Note: <s> and </s> are starting and ending tokens

Using the bigram model, what's the probability of the sentence "<s> I like strawberries </s>"? Ignore the probability of <s>.

(A) 4/9 (B) 1/3 (C) 2/9 (D) 1/9

Estimating probabilities

Consider the following corpus

<s> I like apples </s>

<s> You like strawberries </s>

<s> You like apples </s>

Note: <s> and </s> are starting and ending tokens

Using the bigram model, what's the probability of the sentence "<s> I like strawberries </s>"? Ignore the probability of <s>.

(A) 4/9 (B) 1/3 (C) 2/9 (D) 1/9

P(<s> I like strawberries </s>) $= \frac{1}{3} \cdot 1 \cdot \frac{1}{3} \cdot 1$

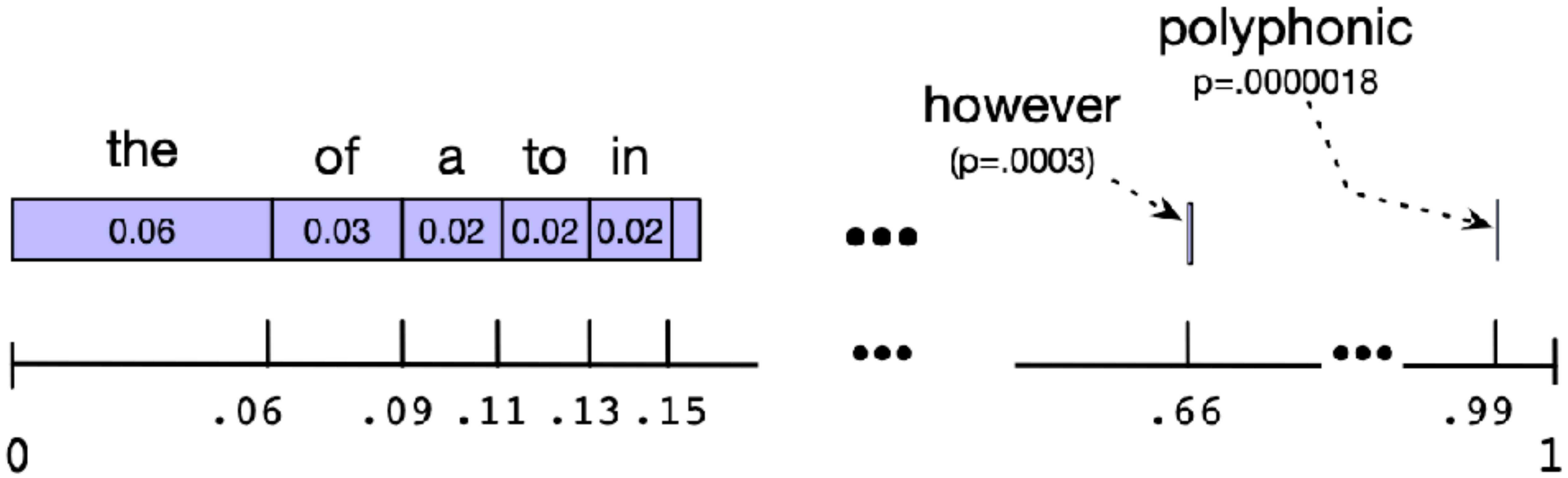
Generating from a language model

Generating from a language model

- Given a language model, how to generate a sequence?

$$\text{Bigram} \quad P(w_1, w_2, ...w_n) = \prod_{i=1}^{n} P(w_i | w_{i-1})$$

- Generate the first word $w_1 \sim P(w)$
- Generate the second word $w_2 \sim P(w \mid w_1)$
- Generate the third word $w_3 \sim P(w \mid w_2)$
- …



Generating from a language model

- Given a language model, how to generate a sequence?

Trigram $$P(w_1, w_2, \ldots, w_n) = \prod_{i=1}^{n} P(w_i \mid w_{i-2}, w_{i-1})$$

- Generate the first word $w_1 \sim P(w)$
- Generate the second word $w_2 \sim P(w \mid w_1)$
- Generate the third word $w_3 \sim P(w \mid w_1, w_2)$
- Generate the fourth word $w_4 \sim P(w \mid w_2, w_3)$
- …

Generations

Unigram

- *To him swallowed confess hear both. Which. Of save on trail for are ay device and rote life have*

Bigram

- *Why dost stand forth thy canopy, forsooth; he is this palpable hit the King Henry. Live king. Follow.*
- *What means, sir. I confess she? then all sorts, he is trim, captain.*

Trigram

- *Fly, and will rid me these news of price. Therefore the sadness of parting, as they say,*
- *This shall forbid it should be branded, if renown made it empty*

Typical LMs are not sufficient to handle long-range dependencies

"The woman/man could not go to work that day because she/he had a doctor's appointment"

Generations

Example from a GPT-2 output (2019):

prompt aka. conditional context

With the start of the new academic year, Princeton has an opportunity to help provide a new generation of women with a diverse set of academic resources for higher education.
We are offering the resources of the Princeton-McGill program specifically to women with undergraduate degrees who would like to enhance their academic experience. Princeton-McGill offers a comprehensive suite of services for women and their families including a variety of graduate programs, support programs, and the opportunity to serve as leaders in their communities with a wide variety of programs, activities and services. For the upcoming fall, Princeton-McGill will also offer its Women's Center , which is located in a renovated women's dorm.
At Princeton, we are working with the Princeton-McGill community to develop a suite of programs that are designed to give new and returning students a strong foundation for a successful, rewarding graduate career. The Women's Center , the Princeton-McGill Women's Center provides a range of supports to address the specific needs of female doctoral degree graduates. Programs are tailored to meet the unique needs of women under the age of 28, women and families

https://talktotransformer.com/

$$P(w_1, w_2, \ldots, w_n) = \prod_{i=1}^{n} P(w_i \mid w_{i-1024}, \ldots, w_{i-2}, w_{i-1})$$

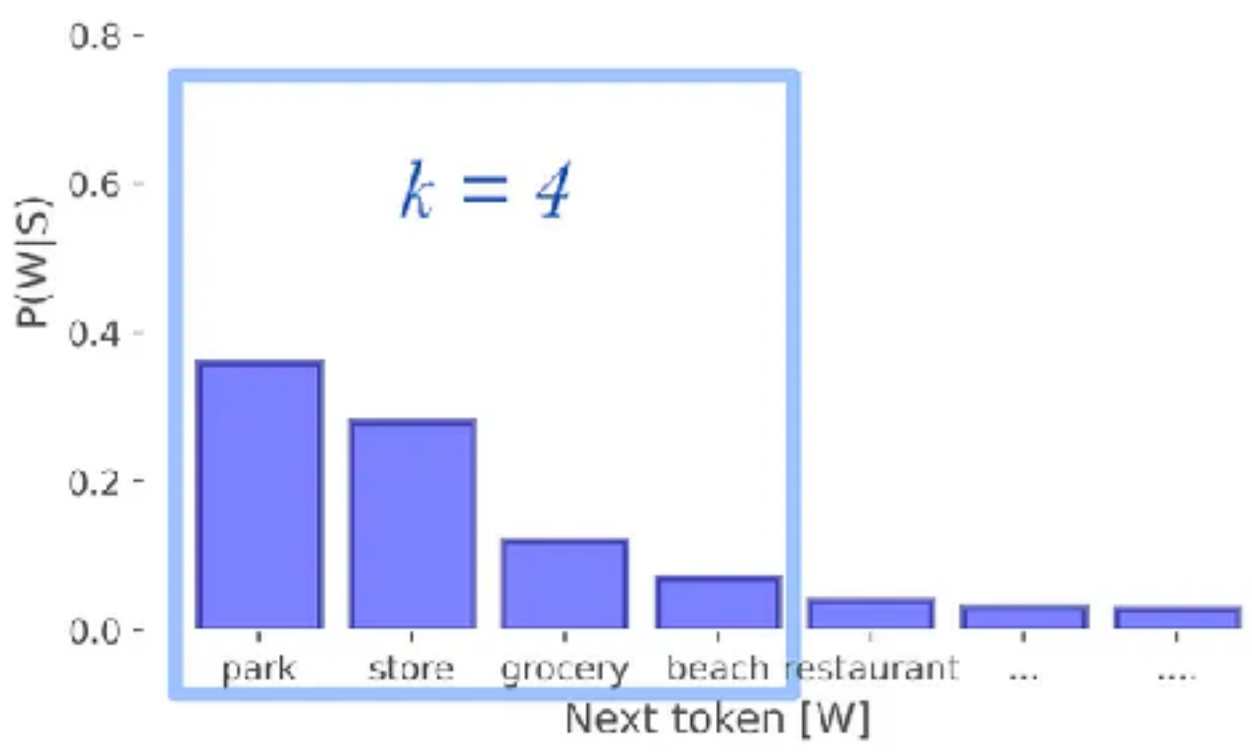
Modern LMs can handle much longer contexts!

Generation methods (advanced)

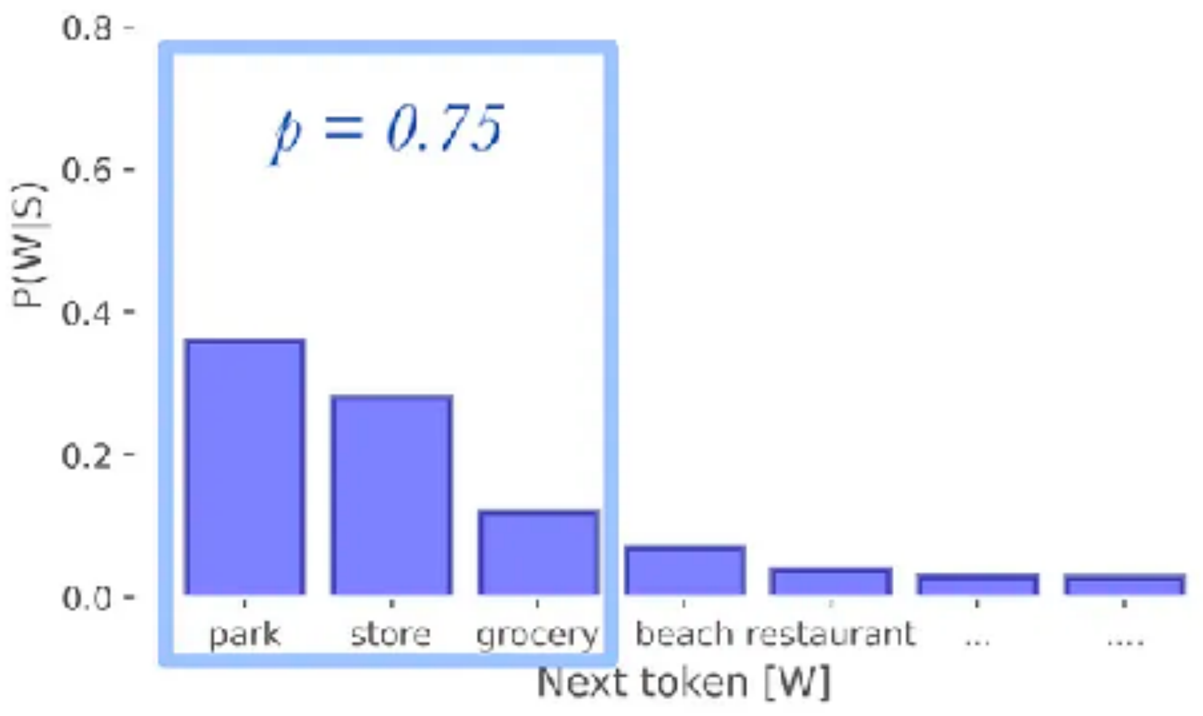
- Greedy: choose the most likely word!
  - To predict the next word given a context of two words w_1, w_2:

$$w_3 = \arg\max_{w \in V} P(w \mid w_1, w_2)$$

- Top-k vs top-p sampling: "The boy went to the _______"



Top-k sampling

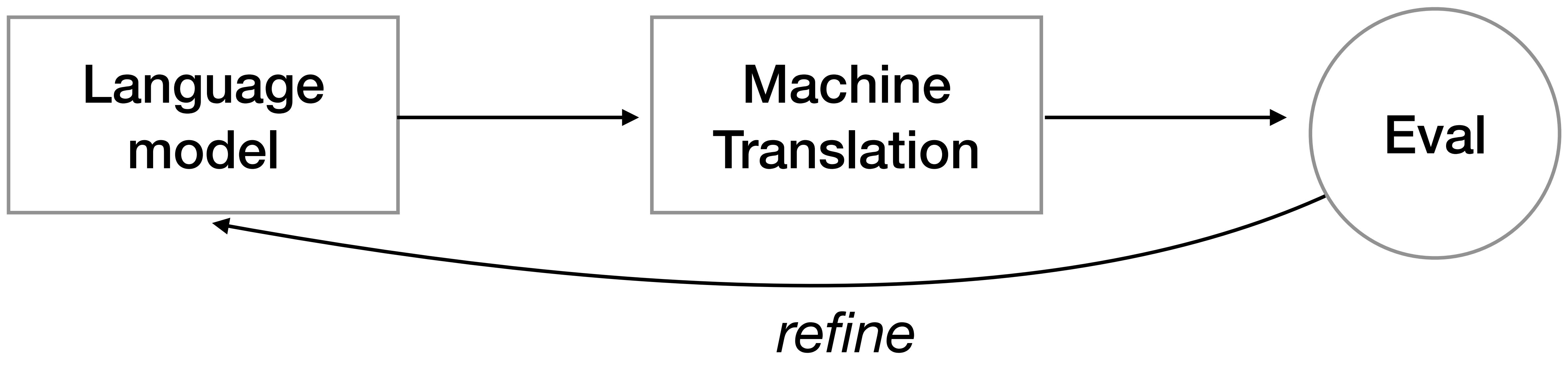


Top-p sampling

https://blog.allenai.org/a-guide-to-language-model-sampling-in-allennlp-3b1239274bc3

Evaluating a language model

Extrinsic evaluation



- Train LM → apply to task → observe accuracy
- Directly optimized for downstream applications
  - higher task accuracy → better model
- Expensive, time consuming
- Hard to optimize downstream objective (indirect feedback)

Intrinsic evaluation of language models

- **Train** parameters on a suitable training corpus
  - Assumption: observed sentences ~ good sentences
- **Test** on different, unseen corpus
  - If a language model assigns a higher probability to the test set, it is better
- Evaluation metric - **perplexity**!

Motivation: Shannon game

Goal for language models: model $\Pr[w_1 w_2 \ldots w_k]$ or $\Pr[w_k \,|\, w_1 \ldots w_{k-1}]$ well.

Shannon game: How well can we predict the next word?

- **I always order pizza with cheese and ____**
- The 33rd president of the US was ____
- I saw a ____

(Slide credit: COS324, Ruth Fong)

Motivation: Shannon game

Goal for language models: model $\Pr[w_1 w_2 \ldots w_k]$ or $\Pr[w_k \mid w_1 \ldots w_{k-1}]$ well.

Shannon game: How well can we predict the next word?

- **I always order pizza with cheese and ____**
- The 33rd president of the US was ____
- I saw a ____

mushrooms 0.1
pepperoni 0.1
anchovies 0.01
…
friend rice 0.0001
…

How would a unigram model do?

(Slide credit: COS324, Ruth Fong)

Motivation: Shannon game

Goal for language models: model $\Pr[w_1 w_2 \dots w_k]$ or $\Pr[w_k \mid w_1 \dots w_{k-1}]$ well.

Shannon game: How well can we predict the next word?

- **I always order pizza with cheese and ____**
- The 33rd president of the US was ____
- I saw a ____

mushrooms 0.1
pepperoni 0.1
anchovies 0.01
…
friend rice 0.0001
…

How would a unigram model do?
Not well, it would assign the most probability to the most common word (i.e. the word that occurs the most in the training corpus).

Better language model = assigns higher probability to word that actually occurs.

(Slide credit: COS324, Ruth Fong)

Perplexity (ppl)

- Measure of how well a LM **predicts** the next word
- For a test corpus with words $w_1, w_2, \ldots w_n$

$$\boxed{\text{Perplexity} = \; P(w_1, w_2, \ldots, w_n)^{-1/n}}$$

$$\text{ppl}(S) = 2^x \quad \text{where} \quad x = -\frac{1}{n}\log_2 P(w_1, \ldots, w_n) = -\frac{1}{n}\sum_{i=1}^{n}\log_2 P(w_i \mid w_1 \ldots w_{i-1})$$

Cross-Entropy

- Unigram model: $x = -\frac{1}{n}\sum_{i=1}^{n}\log P(w_i)$ (since $P(w_j \mid w_1 \ldots w_{j-1}) \approx P(w_j)$)

- Minimizing perplexity ~ maximizing probability of corpus

Intuition on perplexity

$$\text{ppl}(S) = 2^x \quad \text{where} \quad x = -\frac{1}{n}\sum_{i=1}^{n} \log P(w_i \mid w_1 \ldots w_{i-1})$$

If our k-gram model (with vocabulary V) has following probability:

$$P(w \mid w_{i-k}, \ldots w_{i-1}) = \frac{1}{|V|}, \quad \forall w \in V$$

what is the perplexity of the test corpus?

A) $2^{|V|}$ B) $|V|$ C) $|V|^2$ D) $2^{-|V|}$

Intuition on perplexity

$$\text{ppl}(S) = 2^x \quad \text{where} \quad x = -\frac{1}{n}\sum_{i=1}^{n} \log P(w_i \mid w_1 \dots w_{i-1})$$

If our k-gram model (with vocabulary V) has following probability:

$$P(w \mid w_{i-k}, \dots w_{i-1}) = \frac{1}{|V|}, \qquad \forall w \in V$$

what is the perplexity of the test corpus?

A) $2^{|V|}$ B) $|V|$ C) $|V|^2$ D) $2^{-|V|}$

$$\text{ppl} = 2^{-\frac{1}{n} n \log(1/|V|)} = |V|$$

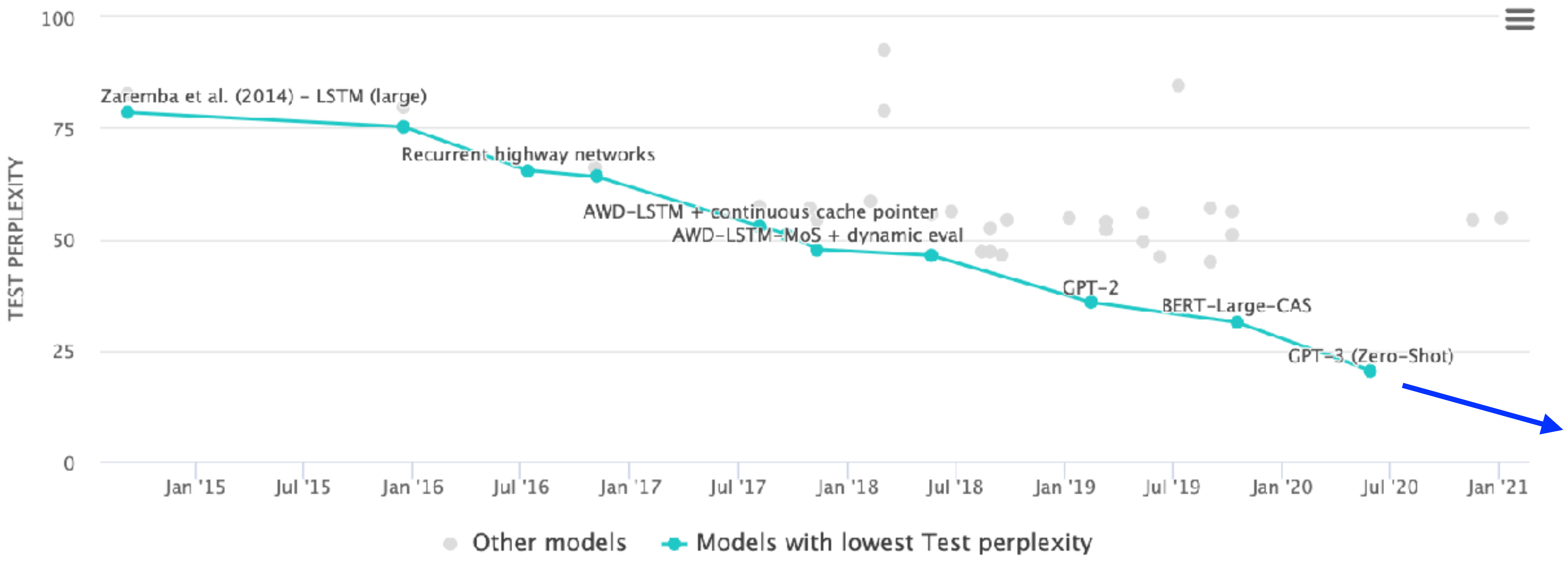
Measure of model's uncertainty about next word (aka `average branching factor')

branching factor = # of possible words following any word

Perplexity

Training corpus 38 million words, test corpus 1.5 million words, both **WSJ**

N-gram Order	Unigram	Bigram	Trigram
Perplexity (test)	962	170	109



GPT-3 175B:
ppl = 20.5

https://paperswithcode.com/sota/language-modelling-on-penn-treebank-word

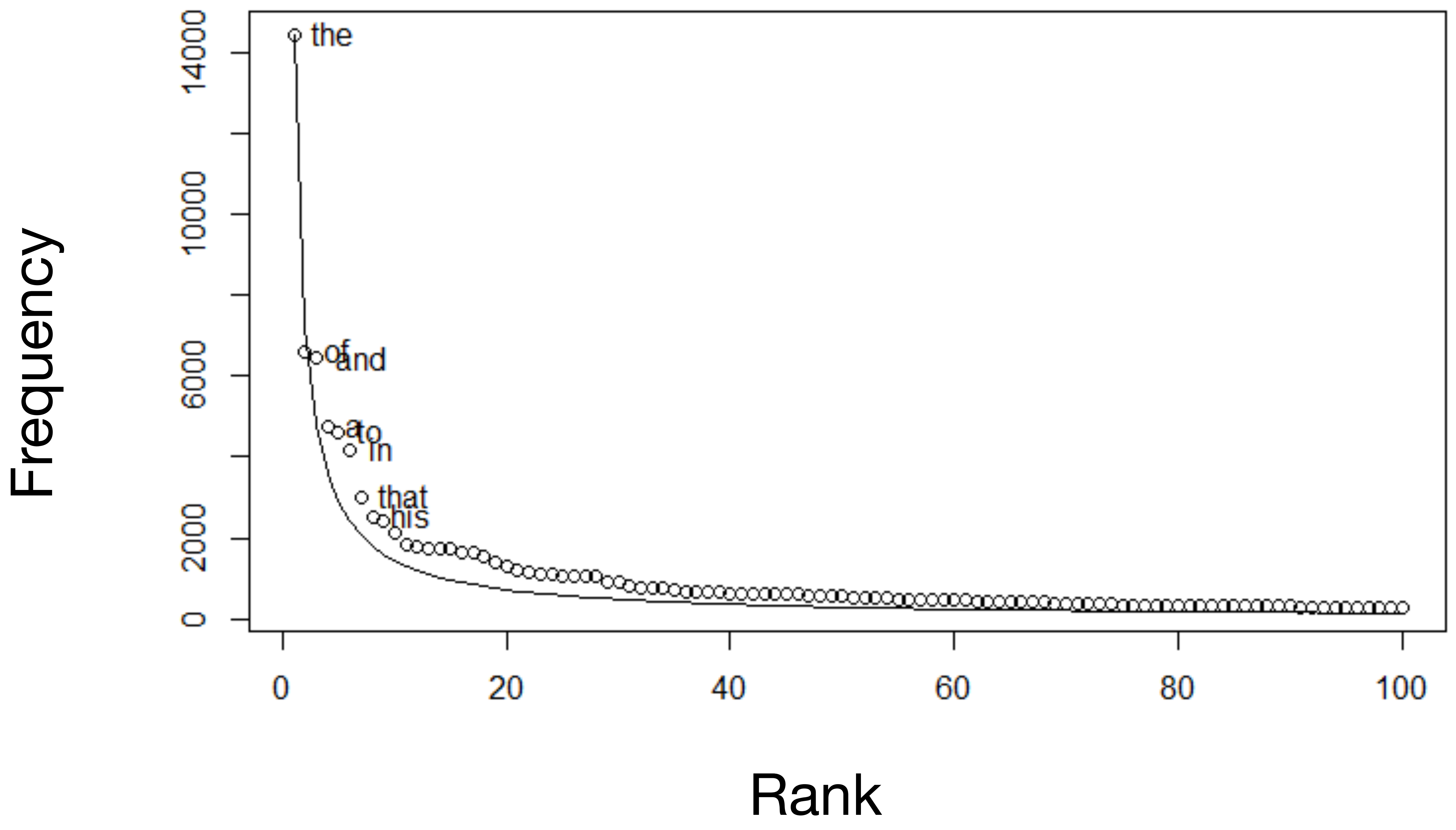
Smoothing

Generalization of n-grams

- Not all n-grams in the test set will be observed in training data
- Test corpus might have some that have zero probability under our model
  - **Training set**: *Google news*
  - **Test set**: *Shakespeare*
  - P(affray | voice doth us) = 0 $\implies$ P(test corpus) = 0
  - Perplexity is not defined.

$$\text{ppl}(S) = 2^x \quad \text{where}$$
$$x = -\frac{1}{n}\sum_{i=1}^{n} \log P(w_i \,|\, w_1 \ldots w_{i-1})$$

Sparsity in language



$$freq \propto \frac{1}{rank}$$

Zipf's Law

- Long tail of infrequent words
- Most finite-size corpora will have this problem.

Smoothing

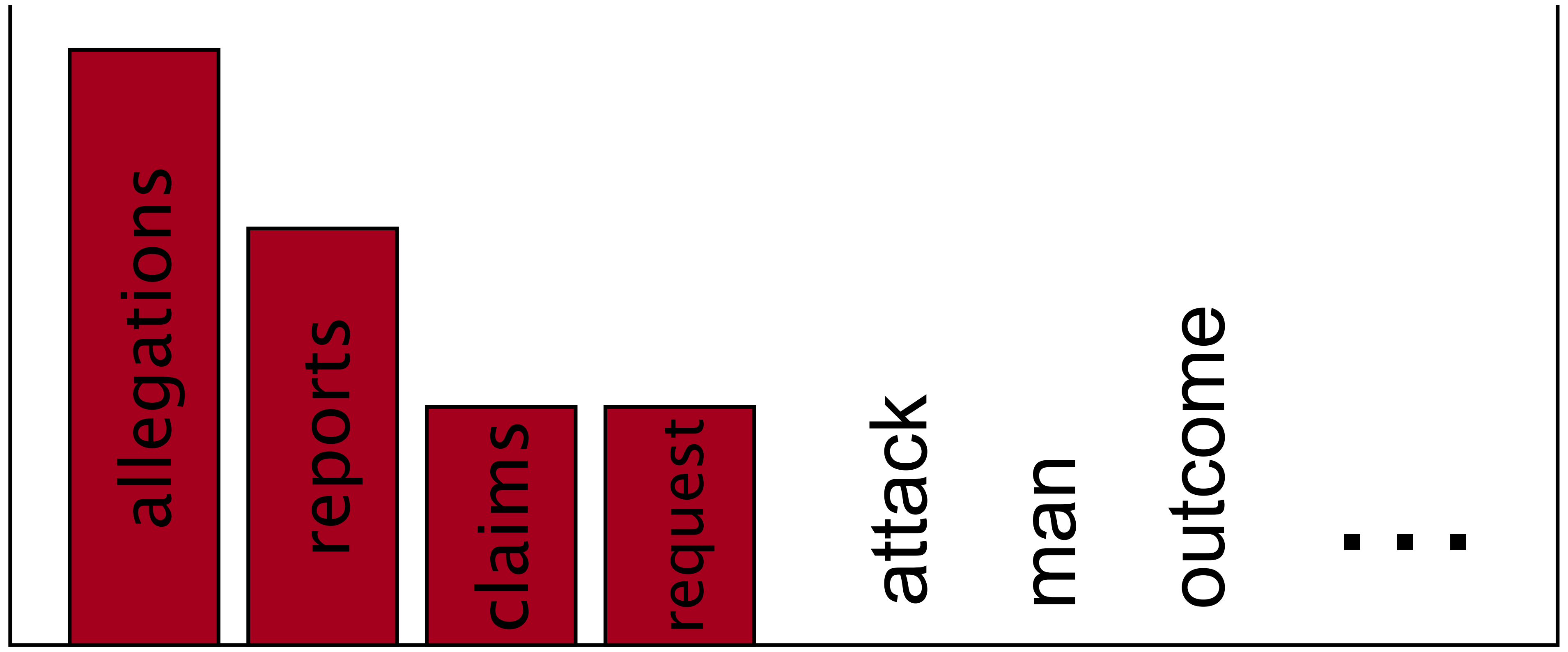
- Handle sparsity by making sure all probabilities are non-zero in our model
  - **Additive**: Add a small amount to all probabilities
  - **Interpolation**: Use a combination of different granularities of n-grams
  - **Discounting**: Redistribute probability mass from observed n-grams to unobserved ones

Smoothing intuition

When we have sparse statistics:

P(w | denied the)
3 allegations
2 reports
1 claims
1 request

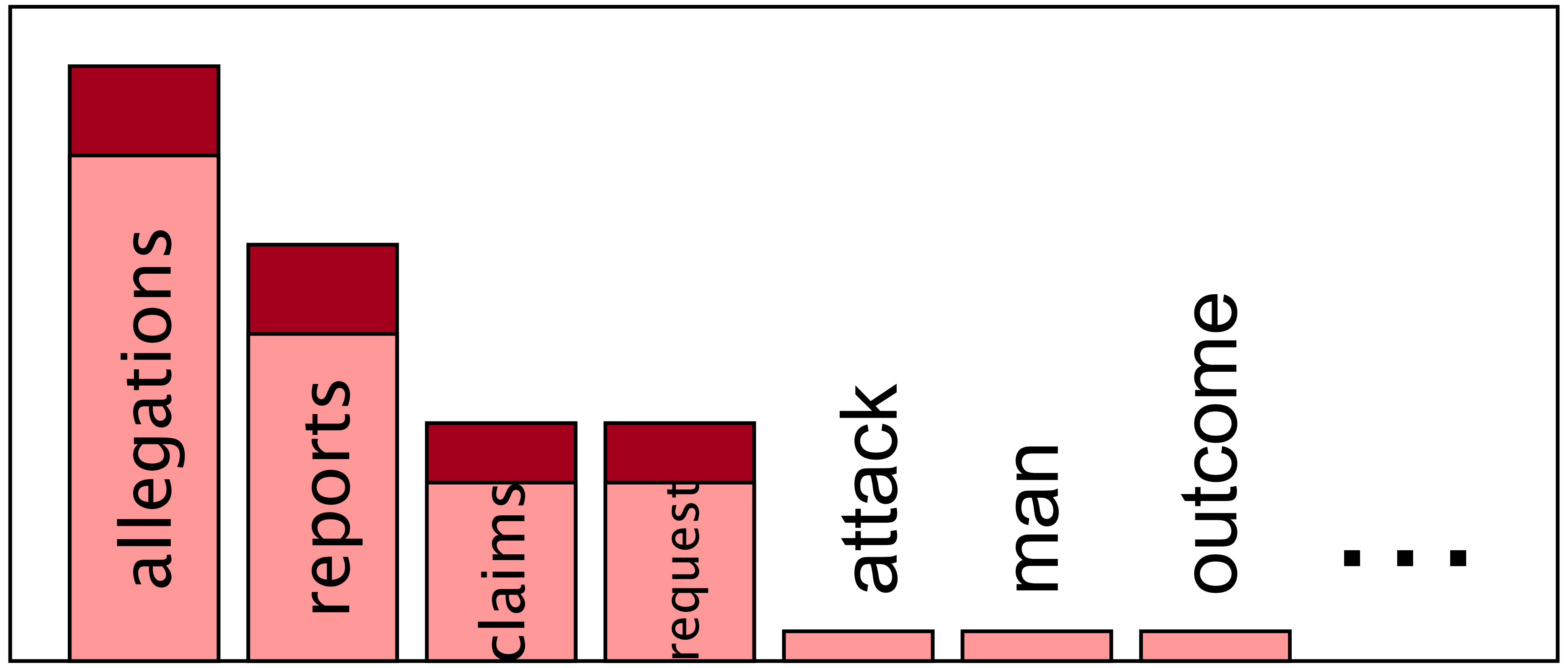
7 total



Steal probability mass to generalize better

P(w | denied the)
2.5 allegations
1.5 reports
0.5 claims
0.5 request
2 other

7 total



(Slide credit: Dan Klein)

Laplace smoothing

- Also known as add-alpha
- Simplest form of smoothing: Just add α to all counts and renormalize!
- Max likelihood estimate for bigrams:

$$P(w_i \mid w_{i-1}) = \frac{C(w_{i-1}, w_i)}{C(w_{i-1})}$$

- After smoothing:

$$P(w_i \mid w_{i-1}) = \frac{C(w_{i-1}, w_i) \boxed{+ \alpha}}{C(w_{i-1}) \boxed{+ \alpha |V|}}$$

Raw bigram counts (Berkeley restaurant corpus)

- Out of 9222 sentences

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

(Slide credit: Dan Jurafsky)

Smoothed bigram counts

	i	want	to	eat	chinese	food	lunch	spend
i	6	828	1	10	1	1	1	3
want	3	1	609	2	7	7	6	2
to	3	1	5	687	3	1	7	212
eat	1	1	3	1	17	3	43	1
chinese	2	1	1	1	1	83	2	1
food	16	1	16	1	2	5	1	1
lunch	3	1	1	1	1	2	1	1
spend	2	1	2	1	1	1	1	1

Add 1 to all the entries in the matrix

(Slide credit: Dan Jurafsky)

Smoothed bigram probabilities

$$P(w_i|w_{i-1}) = \frac{C(w_{i-1}, w_i) \boxed{+ \alpha}}{C(w_{i-1}) \boxed{+ \alpha|V|}} \qquad \alpha = 1$$

	i	want	to	eat	chinese	food	lunch	spend
i	0.0015	0.21	0.00025	0.0025	0.00025	0.00025	0.00025	0.00075
want	0.0013	0.00042	0.26	0.00084	0.0029	0.0029	0.0025	0.00084
to	0.00078	0.00026	0.0013	0.18	0.00078	0.00026	0.0018	0.055
eat	0.00046	0.00046	0.0014	0.00046	0.0078	0.0014	0.02	0.00046
chinese	0.0012	0.00062	0.00062	0.00062	0.00062	0.052	0.0012	0.00062
food	0.0063	0.00039	0.0063	0.00039	0.00079	0.002	0.00039	0.00039
lunch	0.0017	0.00056	0.00056	0.00056	0.00056	0.0011	0.00056	0.00056
spend	0.0012	0.00058	0.0012	0.00058	0.00058	0.00058	0.00058	0.00058

(Credits: Dan Jurafsky)

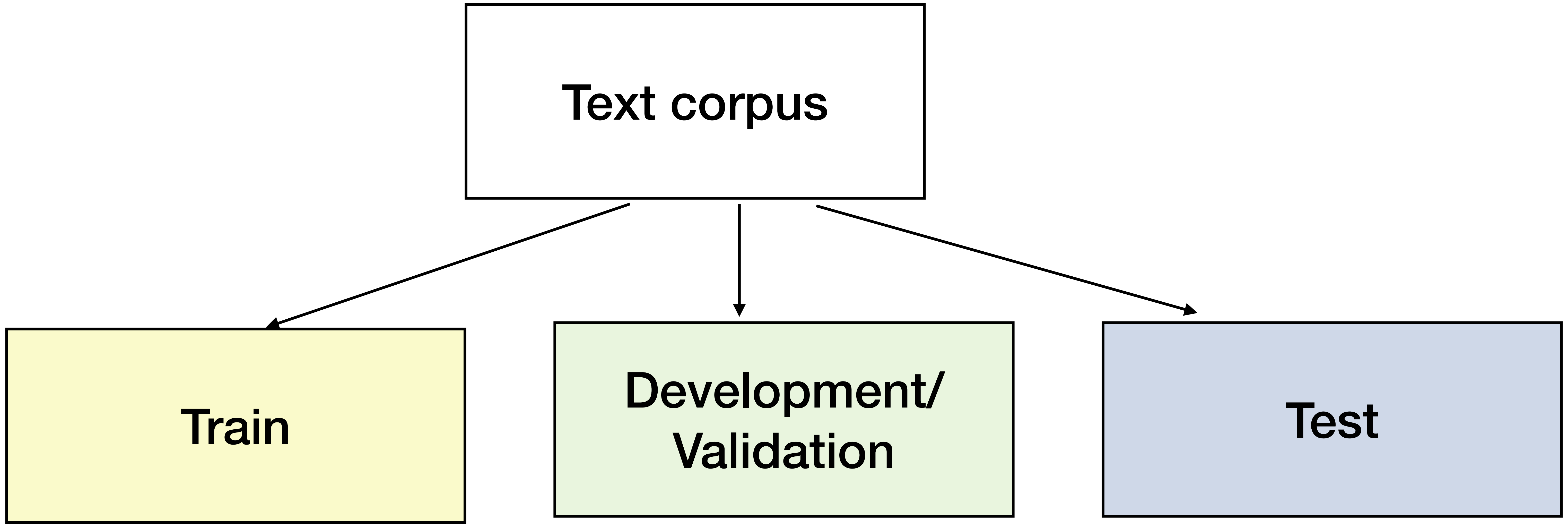
Linear Interpolation

$$\hat{P}(w_i \mid w_{i-2}, w_{i-1}) = \lambda_1 P(w_i \mid w_{i-2}, w_{i-1}) \quad \text{Trigram}$$
$$+\lambda_2 P(w_i | w_{i-1}) \quad \text{Bigram}$$
$$+\lambda_3 P(w_i) \quad \text{Unigram}$$

$$\sum_i \lambda_i = 1$$

- Use a combination of models to estimate probability
- Strong empirical performance

How can we choose lambdas?



- First, estimate n-gram prob. on training set
- Then, estimate lambdas (hyperparameters) to maximize probability on the held-out development/validation set
- Use best model from above to evaluate on test set

Discounting

- Determine some “mass” to remove from probability estimates
- More explicit method for redistributing mass among unseen n-grams
- Just choose an absolute value to discount (usually <1)

Absolute Discounting

- Define Count*(x) = Count(x) - 0.5
- Missing probability mass:

$$\alpha(w_{i-1}) = 1 - \sum_{w} \frac{\text{Count}*(w_{i-1}, w)}{\text{Count}(w_{i-1})}$$

$$\alpha(\text{the}) = 1 - \frac{43}{48} = 5/48$$

- Divide this mass between words w for which Count(the, w) = 0

x	Count(x)	Count*(x)	$\frac{\text{Count}^*(x)}{\text{Count}(x)}$
the	48		
the, dog	15	14.5	14.5/48
the, woman	11	10.5	10.5/48
the, man	10	9.5	9.5/48
the, park	5	4.5	4.5/48
the, job	2	1.5	1.5/48
the, telescope	1	0.5	0.5/48
the, manual	1	0.5	0.5/48
the, afternoon	1	0.5	0.5/48
the, country	1	0.5	0.5/48
the, street	1	0.5	0.5/48

Absolute Discounting

x	Count(x)	Count*(x)	$\frac{\text{Count}^*(x)}{\text{Count}(x)}$
the	48		
the, dog	15	14.5	14.5/48
the, woman	11	10.5	10.5/48
the, man	10	9.5	9.5/48
the, park	5	4.5	4.5/48
the, job	2	1.5	1.5/48
the, telescope	1	0.5	0.5/48
the, manual	1	0.5	0.5/48
the, afternoon	1	0.5	0.5/48
the, country	1	0.5	0.5/48
the, street	1	0.5	0.5/48

$$\alpha(\text{the}) = 10 \times 0.5/48 = 5/48$$

$$P_{\text{abs\_discount}}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i) - d}{c(w_{i-1})} \quad \text{if } c(w_{i-1}, w_i) > 0$$

$$\alpha(w_{i-1}) \frac{P(w_i)}{\sum_{w' \mid c(w_{i-1}, w') = 0} P(w')} \quad \text{if } c(w_{i-1}, w_i) = 0$$

Unigram probabilities

Up next: Text classification