COS 484

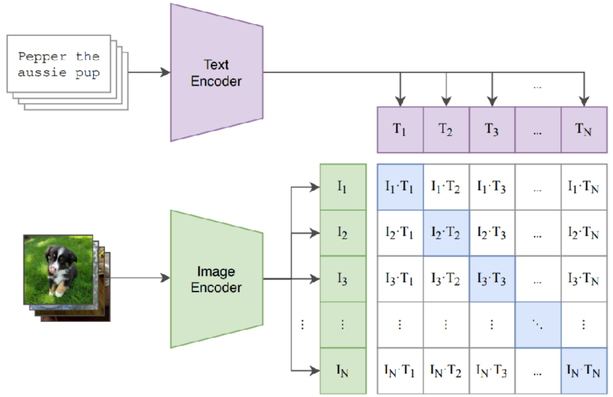
L15: Systems for LLM Training

Spring 2026

Announcements

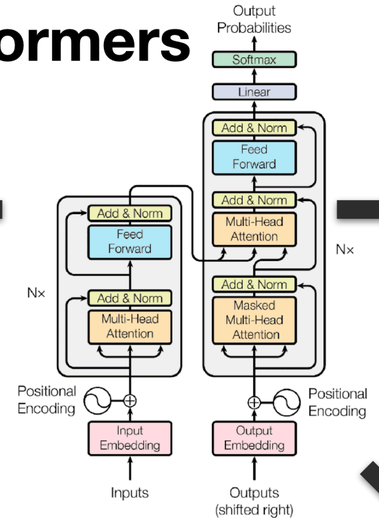
- A3 due today
- A4 will be released later today
- Guest lecture next Tue (April 7): Lei Li (CMU). Zoom only

Transformer models as universal architecture



Natural Language Supervision for Vision (CLIP)

Transformers



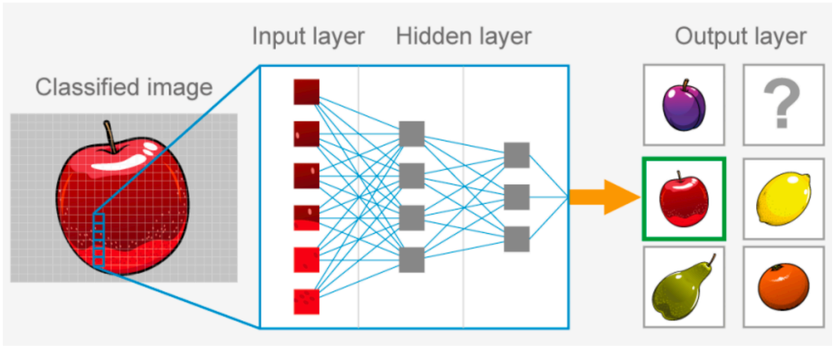
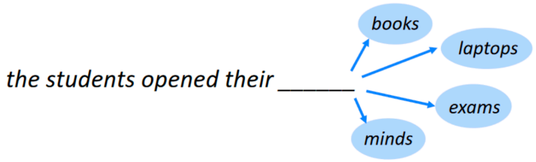


Image Classification (Vision Transformer, Swin-Transformer)



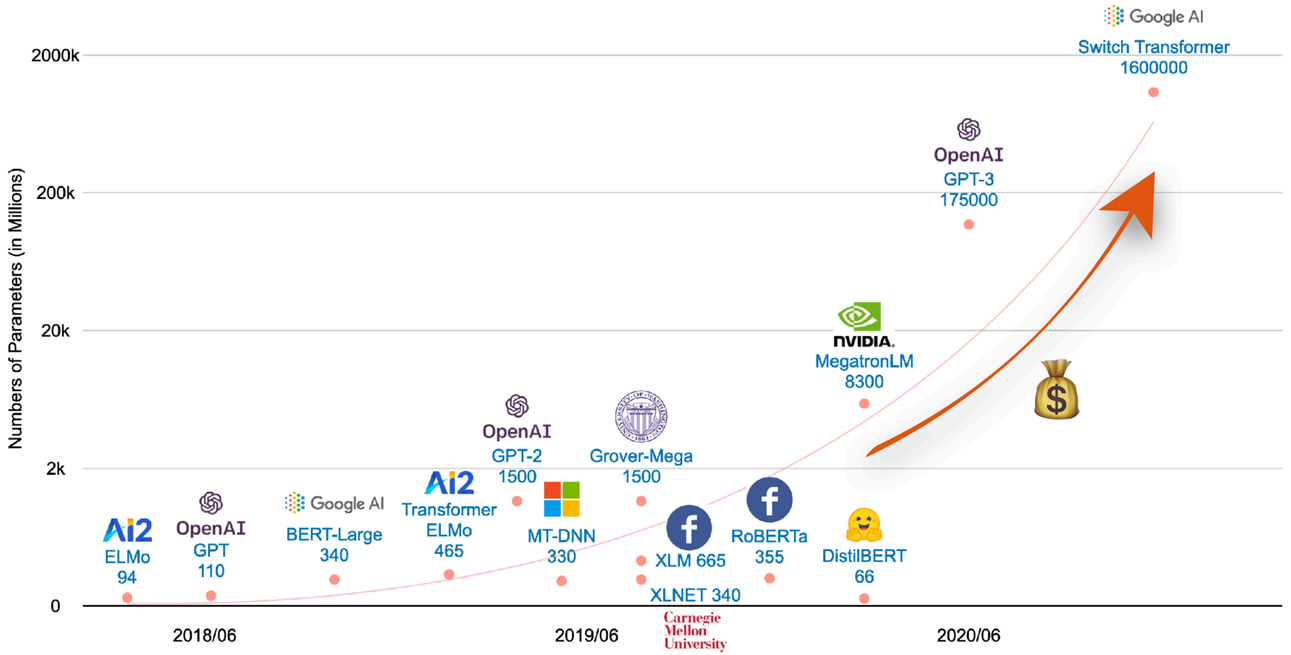
Language Model (BERT, T5, GPT3/4)

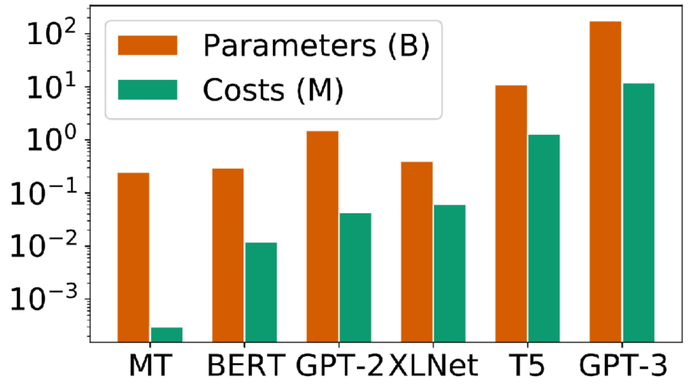
Text to Image (Stable Diffision)

Speech Recognition (wav2vec, HuBERT)

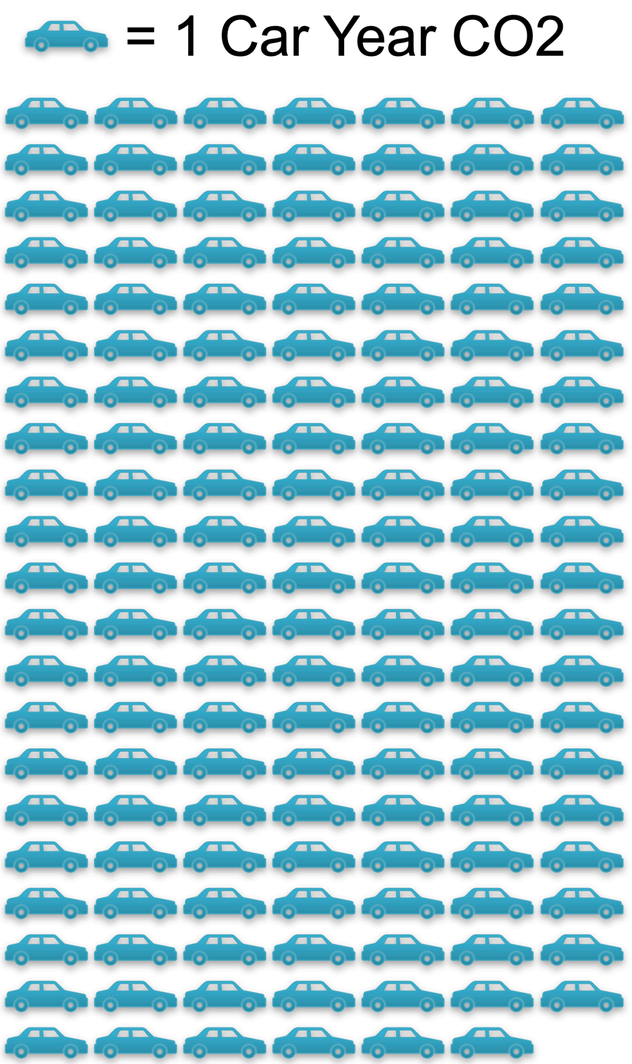
Slide credit: Lei Li

Why we need systems optimization?
LLMs are expensive





Carbon footprint: Training GPT3 = driving a car for 146 years!

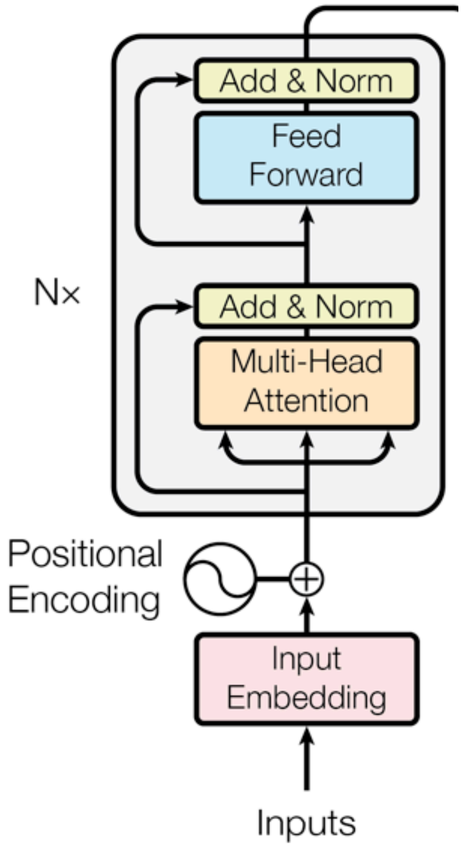


Slide credit: Lei Li

Overview

- (All) Transformer math you need to know
- GPU basics
- Training systems

Transformer architecture



From the bottom to the top:

- Input embedding
- Positional encoding
- A stack of Transformer encoder layers

Transformer encoder is a stack of N layers, which consists of two sub-layers:

- Multi-head attention layer
- Feed-forward layer

$$\mathbf{x}_1, \ldots, \mathbf{x}_n \in \mathbb{R}^{d_1} \longrightarrow \mathbf{h}_1, \ldots, \mathbf{h}_n \in \mathbb{R}^{d_2}$$

Self-attention

$X \in \mathbb{R}^{n \times d_1}$ (n = input length)

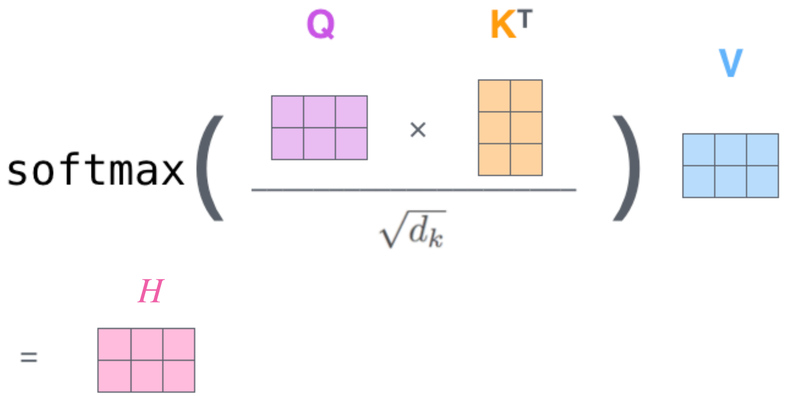
$Q = XW^Q \quad K = XW^K \quad V = XW^V$

$W^Q \in \mathbb{R}^{d_1 \times d_q}, W^K \in \mathbb{R}^{d_1 \times d_k}, W^V \in \mathbb{R}^{d_1 \times d_v}$

$n \times d_q$ $d_k \times n$

$$\text{Attention}(Q, K, V) = \text{softmax}(\frac{QK^T}{\sqrt{d_k}})V$$

$n \times d_v$



https://jalammar.github.io/illustrated-transformer/

Feed-forward Network (MLP)

- There are no elementwise nonlinearities in self-attention; stacking more self-attention layers just re-averages value vectors

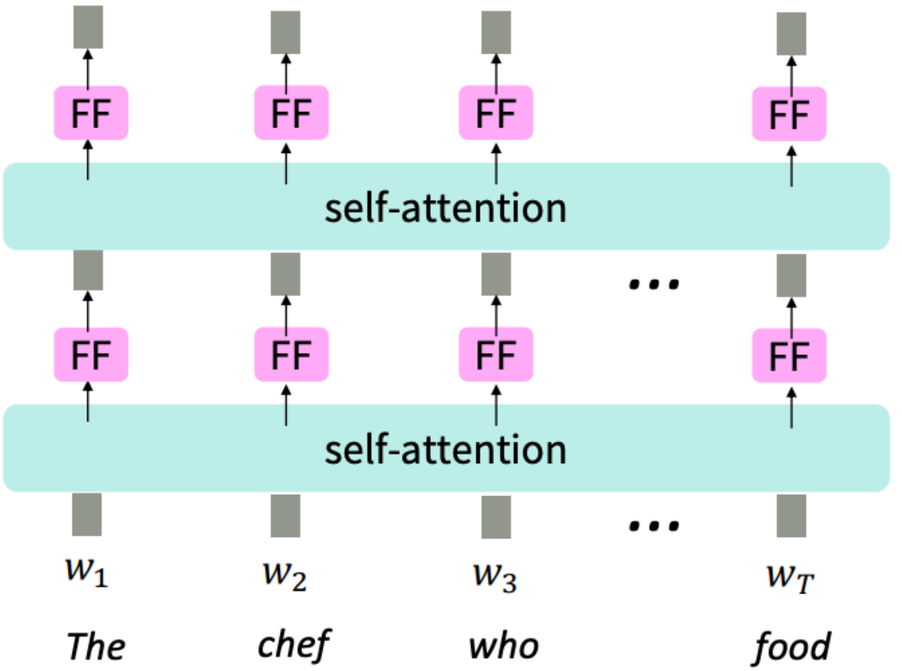
- Simple fix: add a feed-forward network to post-process each output vector

$$\text{FFN}(\mathbf{x}_i) = \text{ReLU}(\mathbf{x}_i\mathbf{W}_1 + \mathbf{b}_1)\mathbf{W_2} + \mathbf{b}_2$$

$$\mathbf{W}_1 \in \mathbb{R}^{d \times d_{ff}}, \mathbf{b}_1 \in \mathbb{R}^{d_{ff}}$$

$$\mathbf{W}_2 \in \mathbb{R}^{d_{ff} \times d}, \mathbf{b}_2 \in \mathbb{R}^{d}$$

In practice, they use $d_{ff} = 4d$



This is actually where the majority of the compute and parameters go!

Residual connection & layer normalization

Add & Norm: $\mathrm{LayerNorm}(x + \mathrm{Sublayer}(x))$

Residual connections (He et al., 2016)

Instead of $X^{(i)} = \mathrm{Layer}(X^{(i-1)})$ (i represents the layer)

We let $X^{(i)} = X^{(i-1)} + \mathrm{Layer}(X^{(i-1)})$, so we only need to learn "the residual" from the previous layer

Gradient through the residual connection is 1 - good for propagating information through layers

Residual connection unlocks deep networks!

Residual connection & layer normalization

Add & Norm: $\mathrm{LayerNorm}(x + \mathrm{Sublayer}(x))$

Layer normalization (Ba et al., 2016) helps train model faster

Idea: normalize the hidden vector values to unit mean and stand deviation within each layer

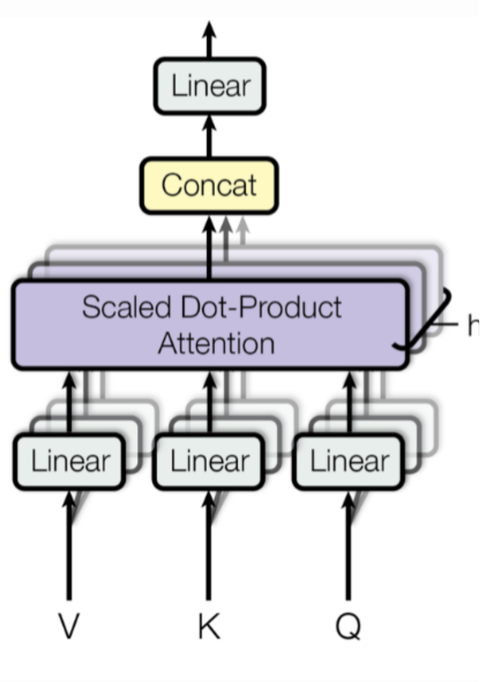
$$y = \frac{x - \mathbf{E}[x]}{\sqrt{\mathbf{Var}[x] + \epsilon}} * \gamma + \beta$$

$\gamma, \beta \in \mathbb{R}^d$ are learnable parameters

LayerNorm is crucial for stable training with deep networks

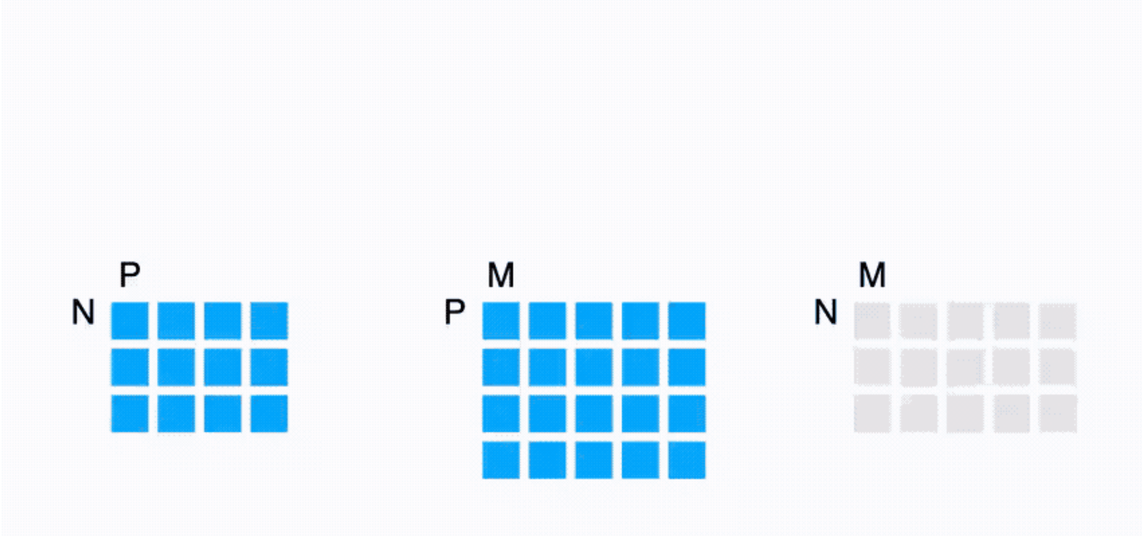
Transformer Math: Parameters

- Embed dim D, number of layers L, vocab size V, context length N
- MHA layer: $4D^2$ ($3D^2$ for QKV projection, D^2 for output projection)
- FFN layer: $8D^2$
  $\text{FFN}(\mathbf{x}_i) = W_2\phi(W_1\mathbf{x}_i + \mathbf{b}_1) + \mathbf{b}_2$
  $\mathbf{W}_1 \in \mathbb{R}^{d \times d_{ff}},\ \mathbf{W}_2 \in \mathbb{R}^{d_{ff} \times d}$
  Typically $d_{ff} = 4d$
- Embedding and LM head: DV each
- Total: $12LD^2 + DV$ if shared embedding, $12LD^2 + 2DV$ if not shared



Transformer Math: Matrix Multiply

- Matmul size (N x P) @ (P x M) has 2MNP FLOPS



- Per input vector, each weight matrix size N x P has 2NP FLOPS

- Forward pass FLOPS per input vector: 2 x no. (non-embedding) params

https://jax-ml.github.io/scaling-book/transformers/

Transformer Math: Forward and Backward FLOPS

- Forward: $A = W\ X$

- Backward:
  Weight gradient $dW = dA\ X^T$
  Activation gradient $dX = W^T\ dA$
  Twice the FLOPS of forward

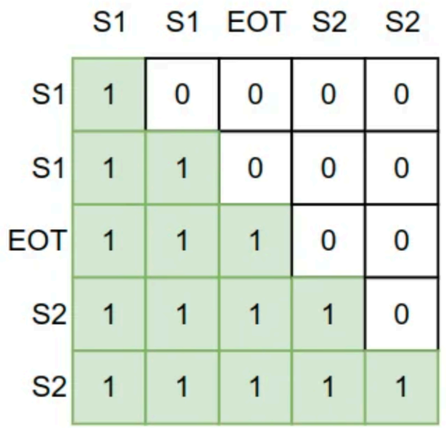
- Forward + backward FLOPS per input vector: 6 x no. (non-embedding) params

Transformer Math: Attention FLOPS

- Forward, for input length N: $4N^2D$ ($2N^2D$ for QK^T, $2N^2D$ for PV)
  N: context length, D: model dimension
  Per input vector: $4ND$

- Backward: 2x forward

- Forward + backward FLOPS per input vector: 12LND
  L: number of layers



- For causal attention: only compute half the entries -> 6LND
  Convention varies on how to count
  (should it be 12LND, 6LND, or even 7LND due to recompute in the backward pass?)

Transformer Math: Total FLOPS

- Forward + backward FLOPS per input vector:
  6 x no. (non-embedding) params + 12LND

  L layers, context length N, model dim D

- Typically approximated as: 6 x no. (non-embedding) params
  when context length is not too long

- Total FLOPS when trained on T tokens: ≈ 6 x no. (non-embedding) params x no. tokens

Overview

- (All) Transformer math you need to know
- **GPU basics**
- Training systems

Hardware

NVIDIA A100 TENSOR CORE GPU SPECIFICATIONS (SXM4 AND PCIE FORM FACTORS)

	A100 80GB PCIe	A100 80GB SXM
FP64	**9.7 TFLOPS**	
FP64 Tensor Core	**19.5 TFLOPS**	
FP32	**19.5 TFLOPS**	
Tensor Float 32 (TF32)	**156 TFLOPS | 312 TFLOPS***	
BFLOAT16 Tensor Core	**312 TFLOPS | 624 TFLOPS***	
FP16 Tensor Core	**312 TFLOPS | 624 TFLOPS***	
INT8 Tensor Core	**624 TOPS | 1248 TOPS***	
GPU Memory	**80GB HBM2e**	**80GB HBM2e**
GPU Memory Bandwidth	**1,935GB/s**	**2,039GB/s**
Max Thermal Design Power (TDP)	**300W**	**400W*** **
Multi-Instance GPU	**Up to 7 MIGs @ 10GB**	**Up to 7 MIGs @ 10GB**
Form Factor	**PCIe dual-slot air cooled or single-slot liquid cooled**	**SXM**
Interconnect	**NVIDIA® NVLink® Bridge for 2 GPUs: 600GB/s ** PCIe Gen4: 64GB/s**	**NVLink: 600GB/s PCIe Gen4: 64GB/s**
Server Options	**Partner and NVIDIA-Certified Systems™ with 1-8 GPUs**	**NVIDIA HGX™ A100-Partner and NVIDIA-Certified Systems with 4,8, or 16 GPUs NVIDIA DGX™ A100 with 8 GPUs**

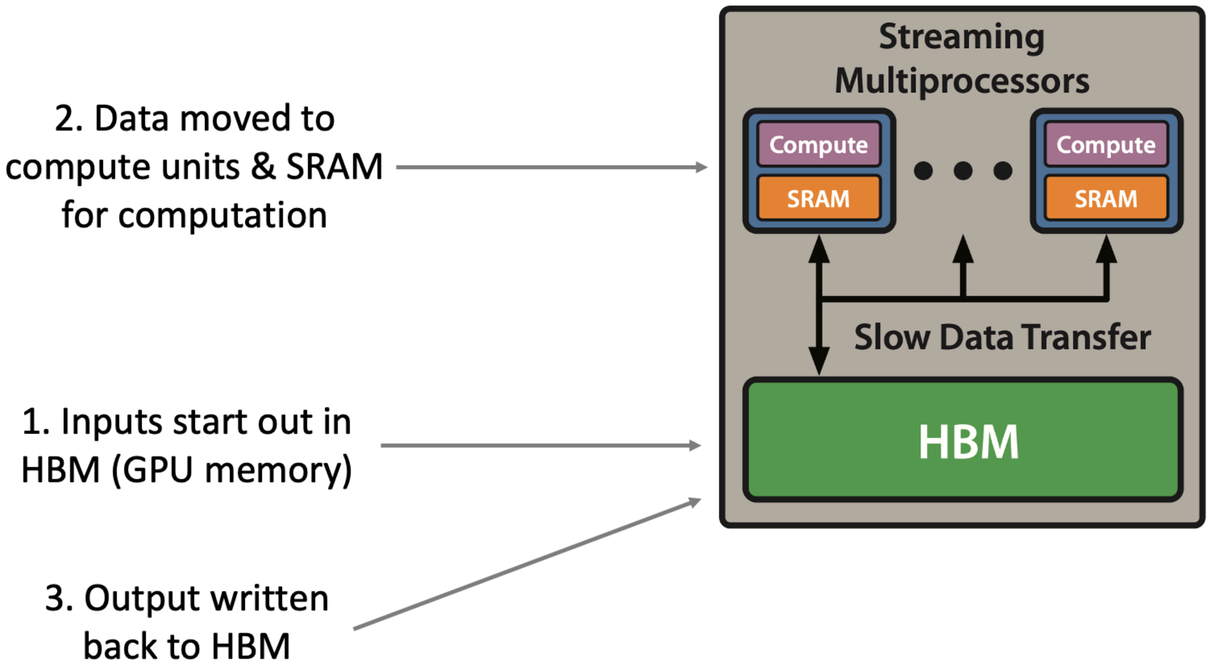
* With sparsity

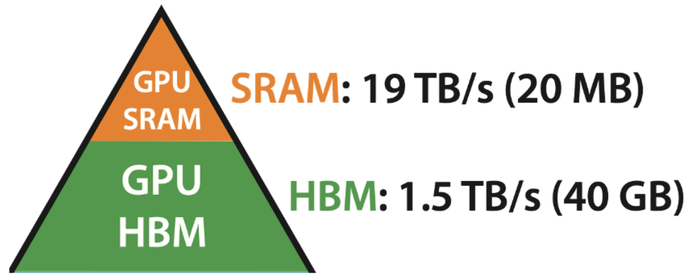
Technical Specifications	H100 SXM
FP64	34 teraFLOPS
FP64 Tensor Core	67 teraFLOPS
FP32	67 teraFLOPS
TF32 Tensor Core*	989 teraFLOPS
BFLOAT16 Tensor Core*	1,979 teraFLOPS
FP16 Tensor Core*	1,979 teraFLOPS
FP8 Tensor Core*	3,958 teraFLOPS
INT8 Tensor Core*	3,958 TOPS
GPU Memory	80GB
GPU Memory Bandwidth	3.35TB/s
Decoders	7 NVDEC 7 JPEG
Max Thermal Design Power (TDP)	Up to 700W (configurable)
Multi-Instance GPUs	Up to 7 MIGs @ 10GB each
Form Factor	SXM
Interconnect	**NVIDIA NVLink™:** 900GB/s **PCIe Gen5:** 128GB/s
Server Options	NVIDIA HGX H100 Partner and NVIDIA-Certified Systems™ with 4 or 8 GPUs NVIDIA DGX H100 with 8 GPUs
NVIDIA Enterprise	Add-on

*With sparsity

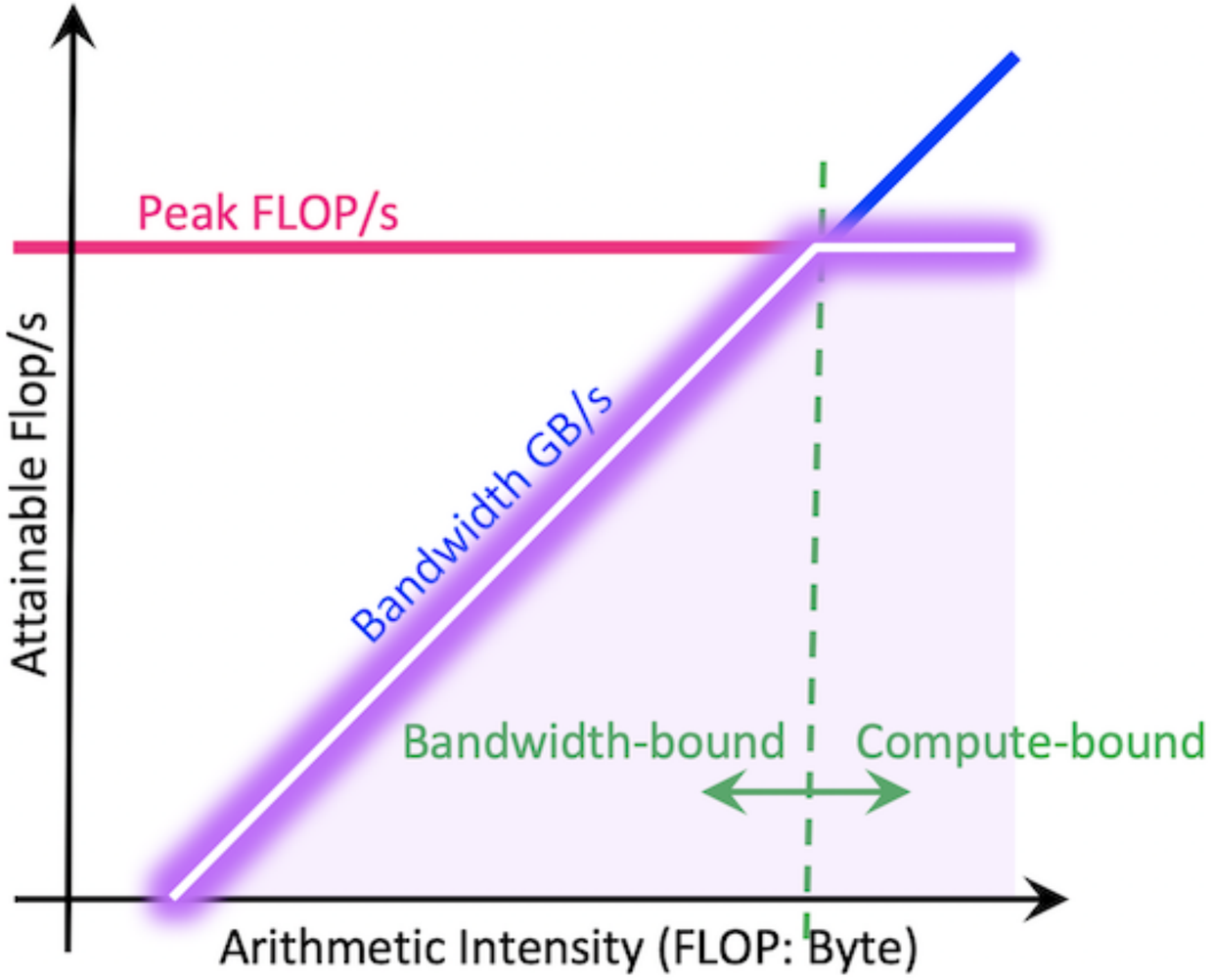
BF16 Dense:
989 TFLOPS

Background: GPU Compute model





Roofline model



How long does it take to train a model?

- Model FLOPS utilization (MFU)
  MFU = Model FLOPS per sec / theoretical max TFLOPS per sec

- Typical MFU: 30-50% (e.g. 300-500 TFLOPS/sec per H100)

- Why: memory-bandwidth bound operations, communication, power-throttling

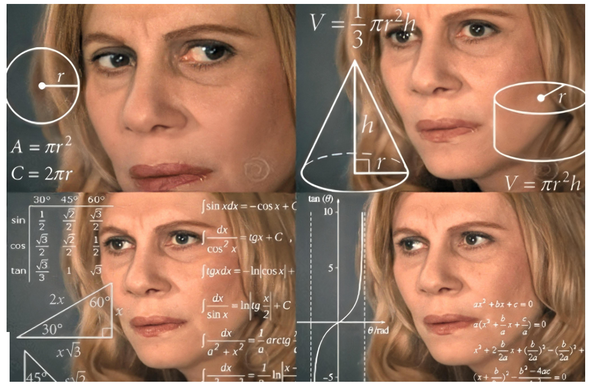
- E.g., how many H100 hours does it take to train a X billion model to Y trillion tokens?
  $\approx 6 * 10^9$X $* 10^{12}$Y FLOPS / ($400 * 10^{12} * 3600$ sec)

How long does it take to train a Llama?

- How many H100 hours does it take to train a X billion model to Y trillion tokens?
  $\approx 6 * 10^9$X $* 10^{12}$Y FLOPS / $(400 * 10^{12} * 3600$ sec)

- E.g., Llama-3: 405B, 15T tokens

  ≈ 6 * 405e9 * 15e12 / (400e12 * 3600)
  = 25.3M hours = 66 days on 16K H100



	Training Time (GPU hours)	Training Power Consumption (W)	Training Location-Based Greenhouse Gas Emissions (tons CO2eq)	Training Market-Based Greenhouse Gas Emissions (tons CO2eq)
Llama 3.1 8B	1.46M	700	420	0
Llama 3.1 70B	7.0M	700	2,040	0
Llama 3.1 405B	30.84M	700	8,930	0
Total	39.3M		11,390	0

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Training: Mixed Precision

Technical Specifications	
	H100 SXM
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BF16 Dense:
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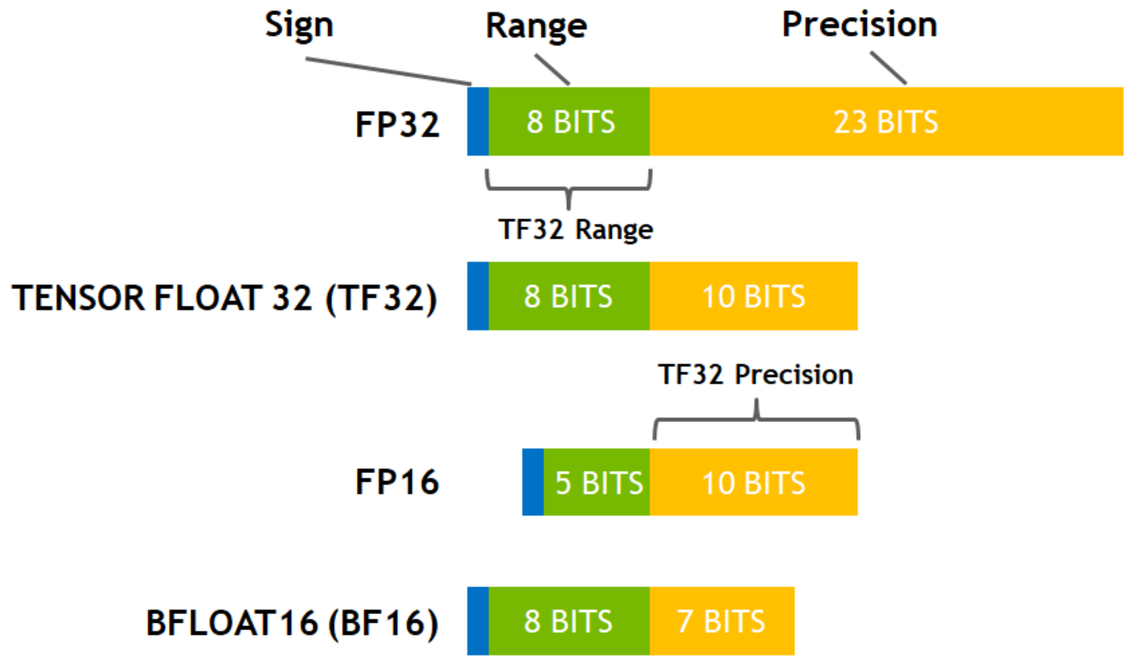


BF16/FP16 FLOPS are much higher than FP32!

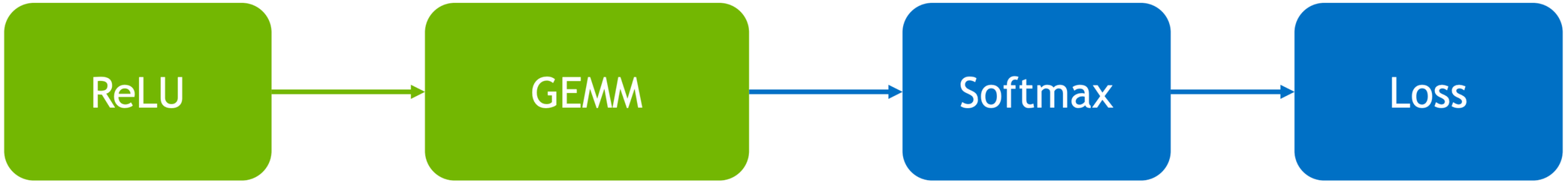
Training: Automatic Mixed Precision

FP16

- GEMMs + Convolutions can use Tensor Cores
- Most pointwise ops (e.g. add, multiply): 1/2X memory storage for intermediates, 2X memory throughput

FP32

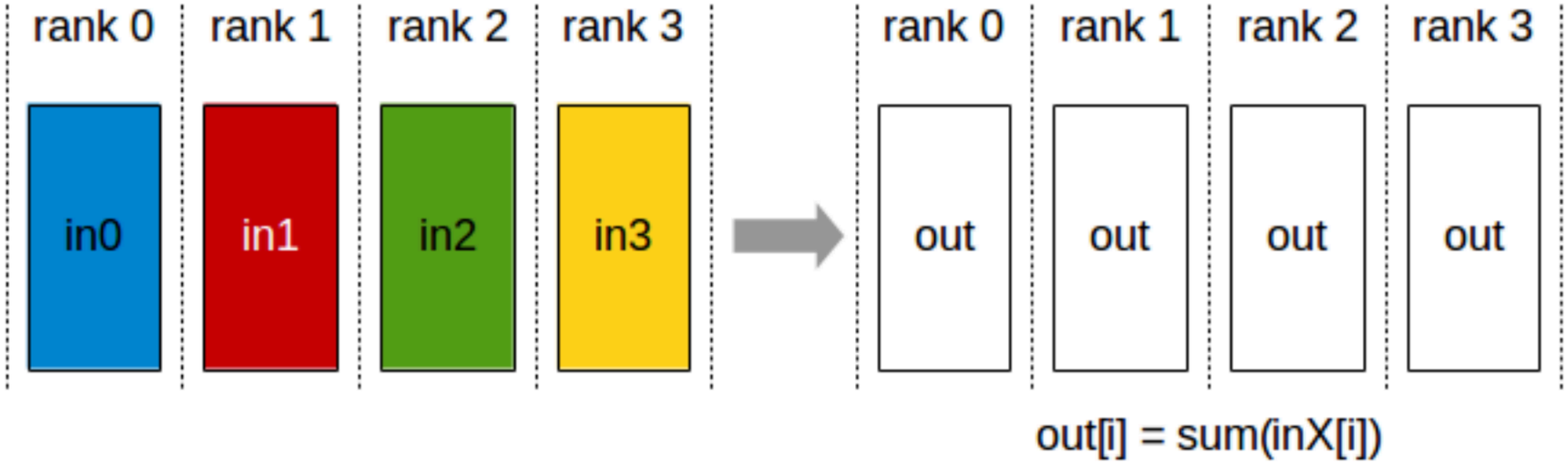
- Weight updates benefit from precision
- Loss functions (often reductions) benefit from precision and range
- Softmax, norms, some other ops benefit from precision and range



Done automatically if you use PyTorch AMP

Distributed Training: Communication AllReduce

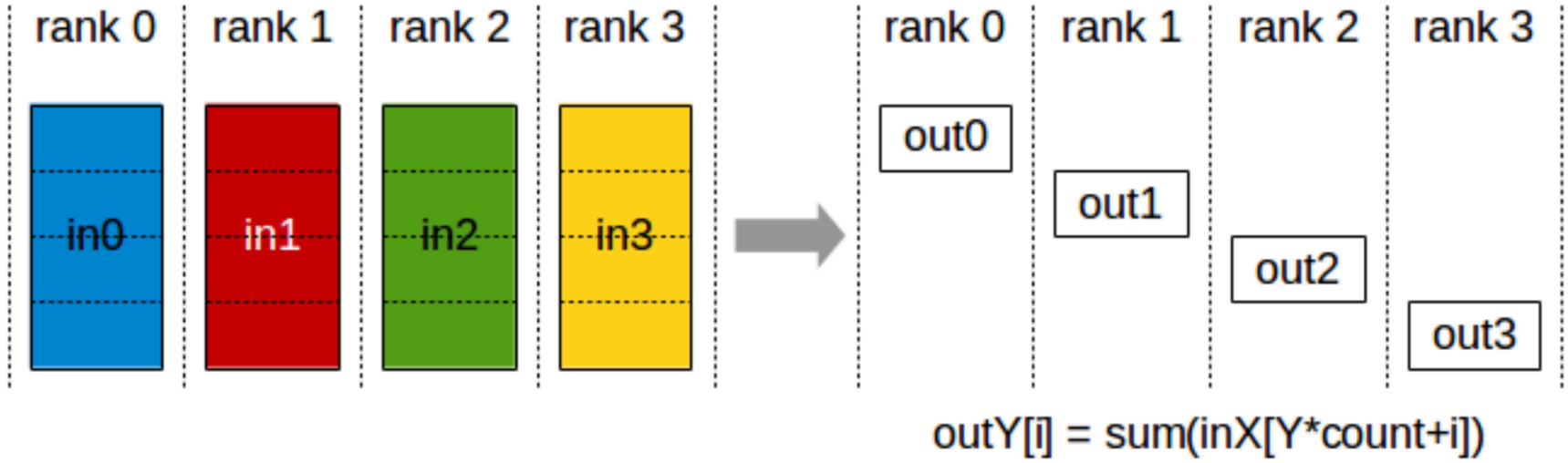
- Compute reduction (sum, min, max) across devices and writing the result in the receive buffers of every rank.



Slide credit: Lei Li

Distributed Training: Communication ReduceScatter

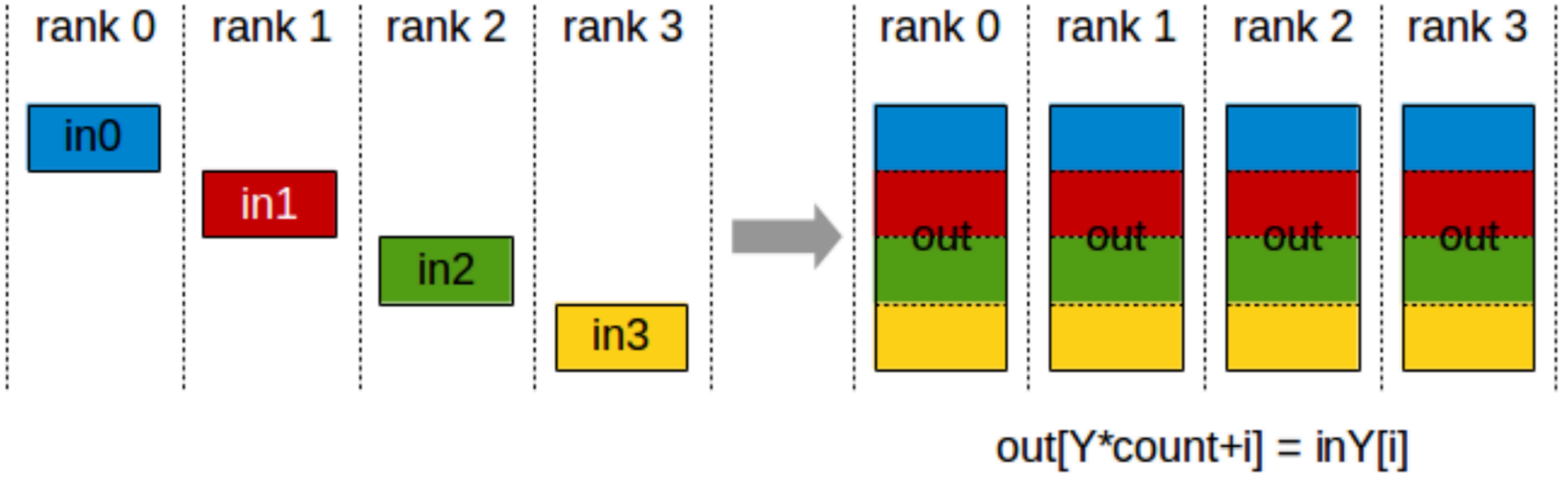
- Compute reduction (sum, min, max) and writing parts of results scattered in ranks



Slide credit: Lei Li

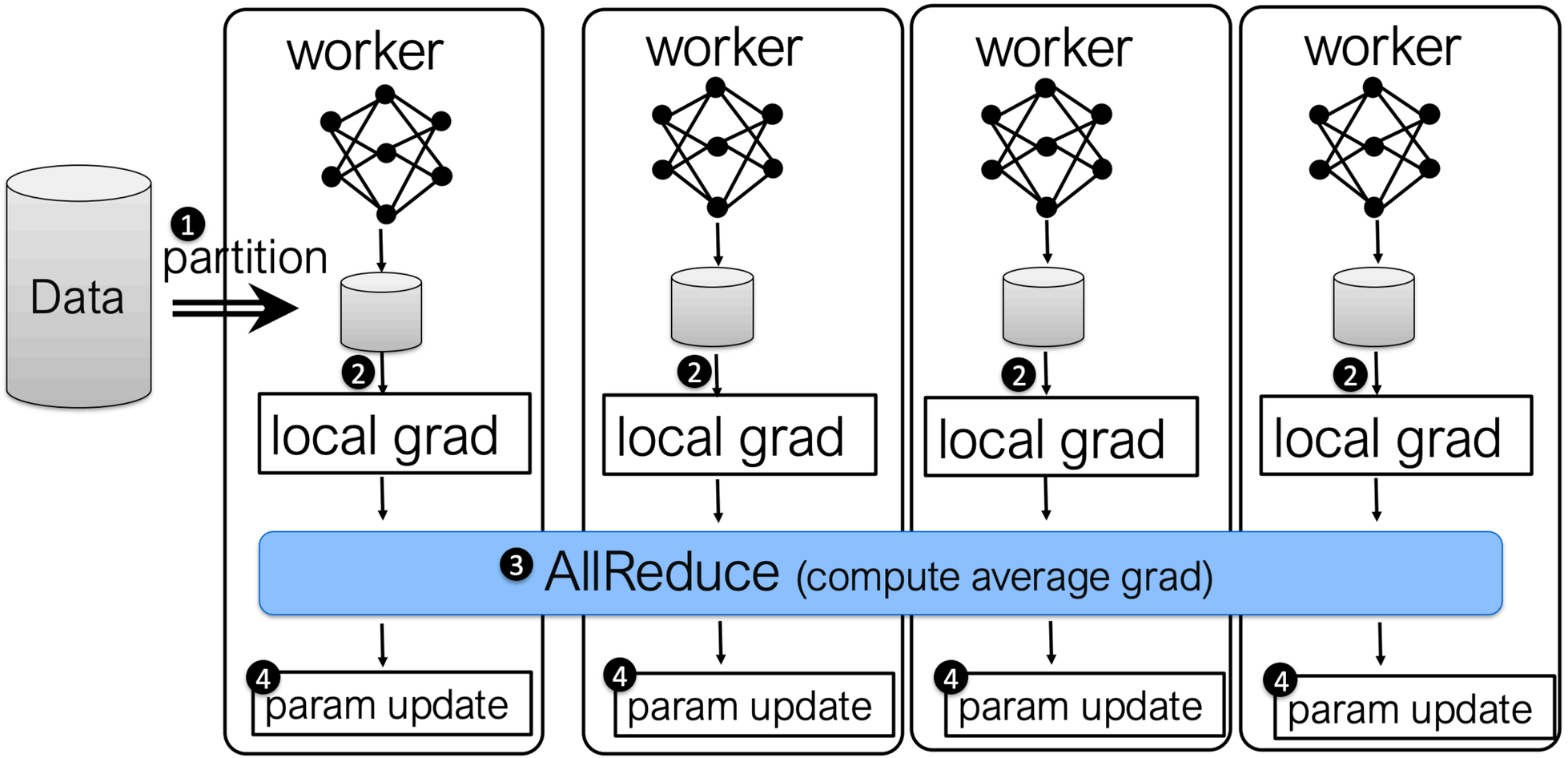
Distributed Training: Communication AllGather

- gathers N values from k ranks into an output of size k*N, and distributes that result to all ranks (devices).



Slide credit: Lei Li

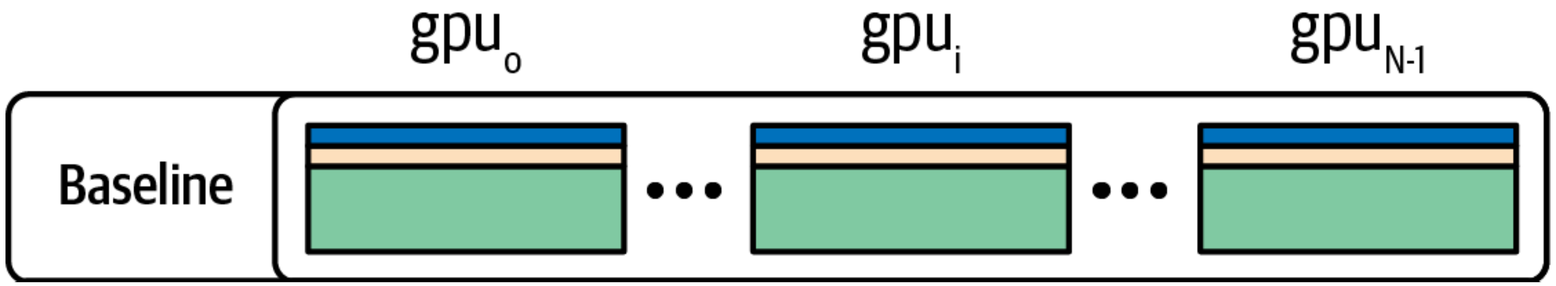
Distributed Training: Data Parallel



PyTorch calls this Distributed Data Parallel (DDP)

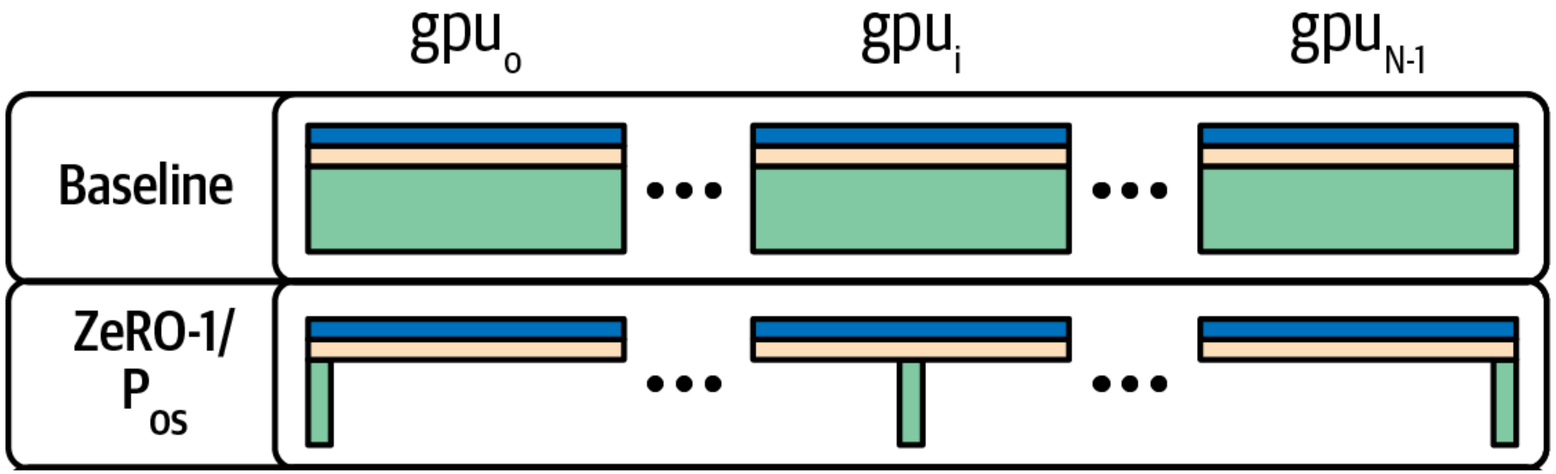
Slide credit: Lei Li

Distributed Training:
Zero Redundancy Optimizer (ZeRO)



PyTorch: Fully Sharded Data Parallel (FSDP)

Distributed Training:
Zero Redundancy Optimizer (ZeRO)



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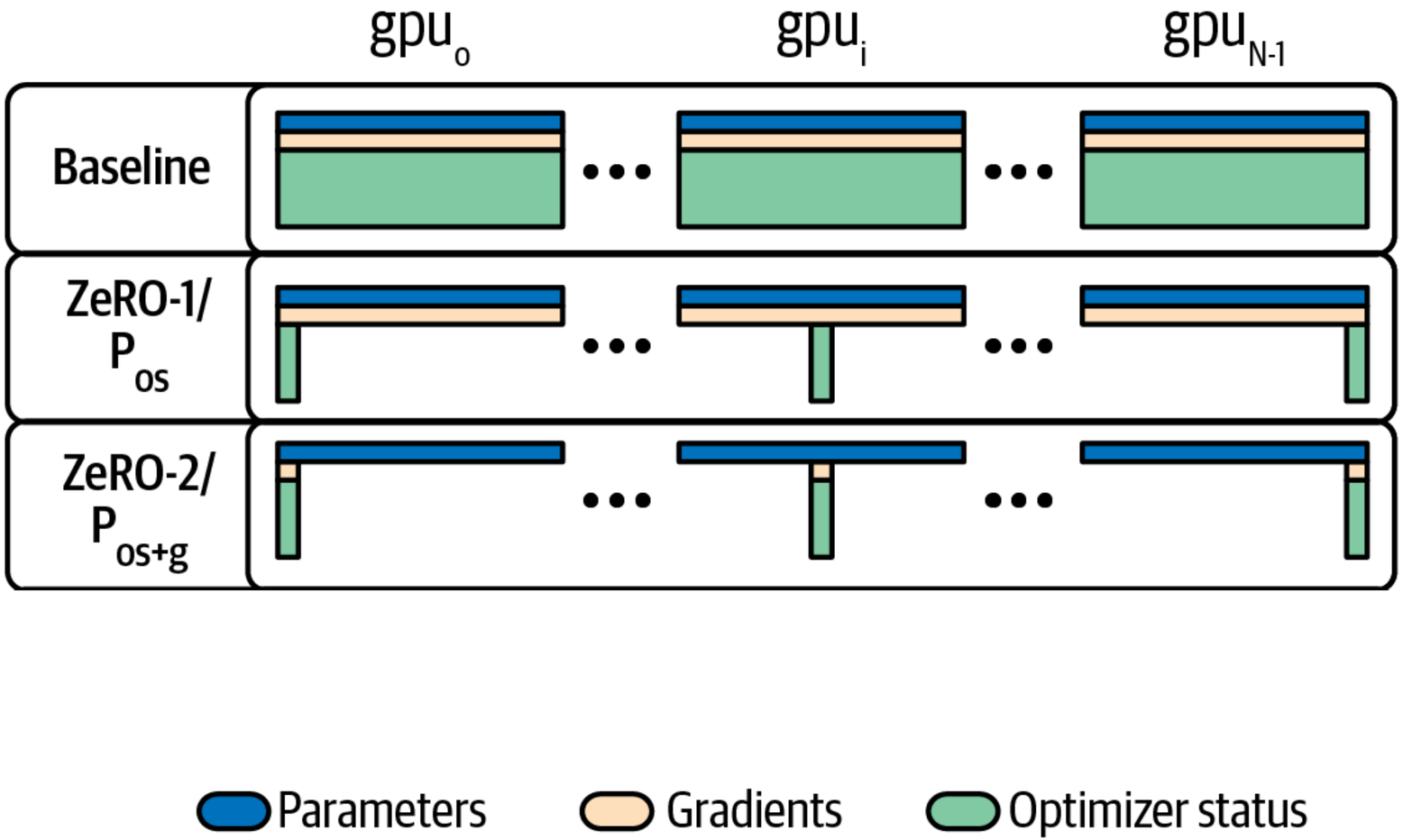
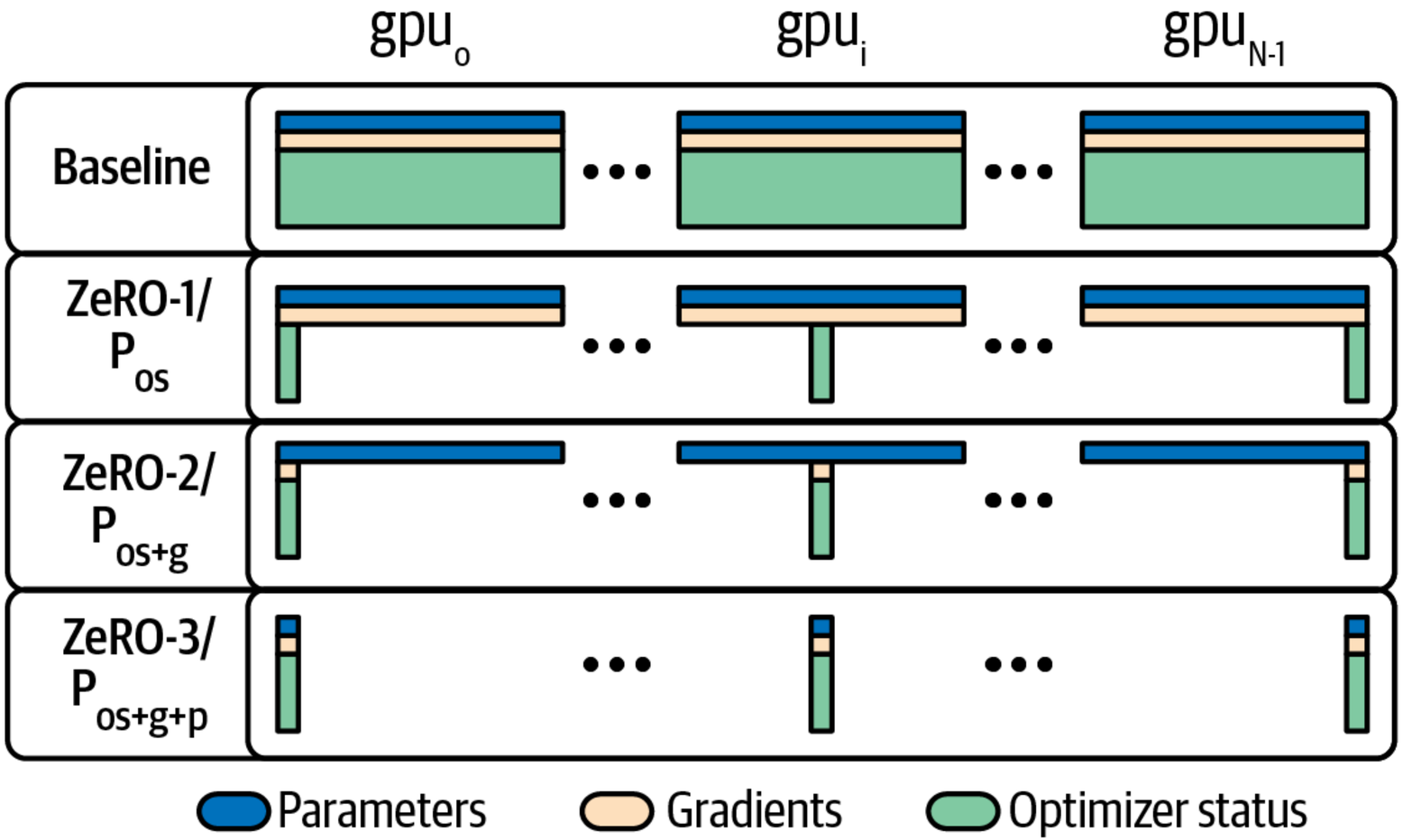


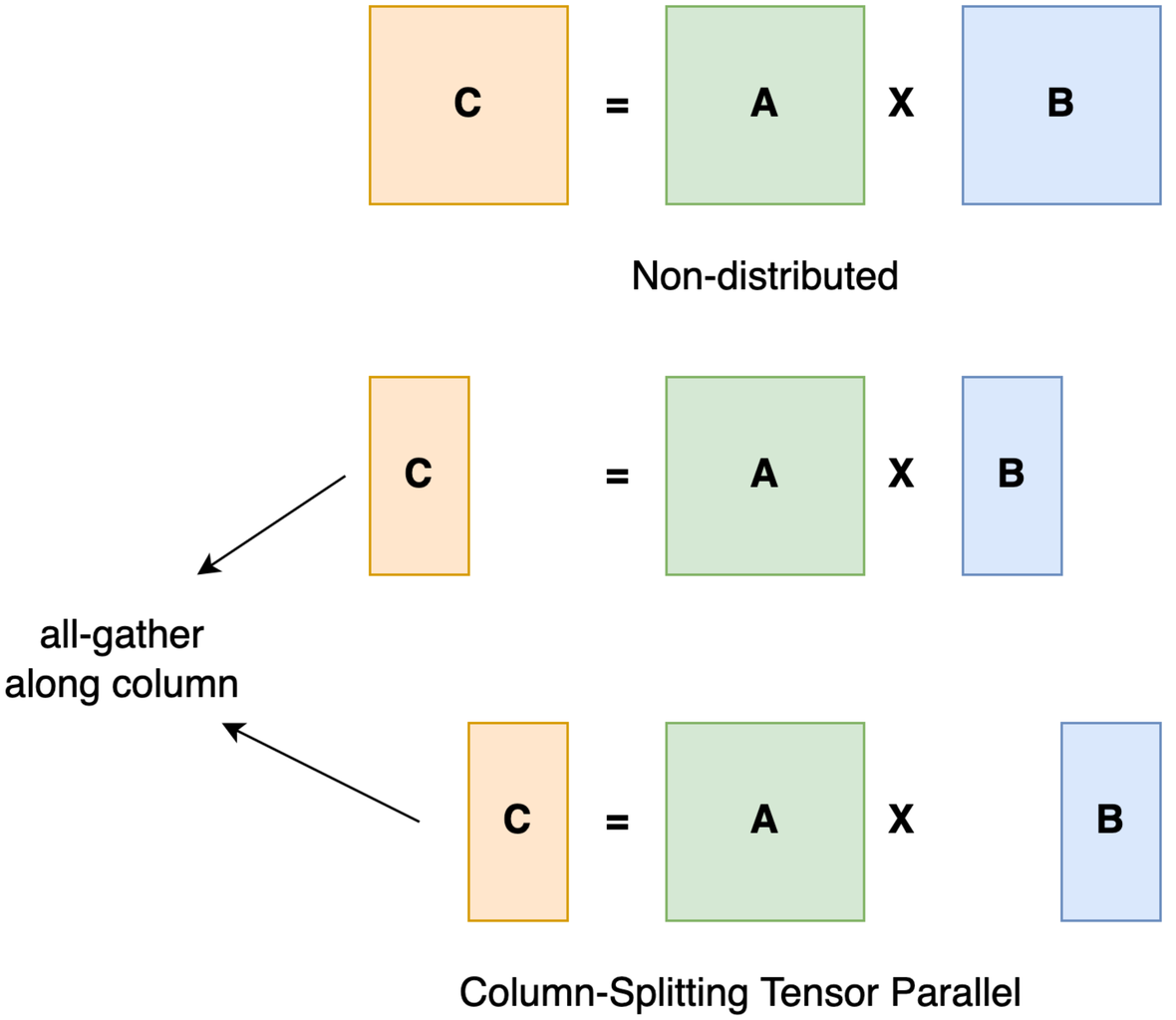
PyTorch: Fully Sharded Data Parallel (FSDP)

Distributed Training:
Zero Redundancy Optimizer (ZeRO)

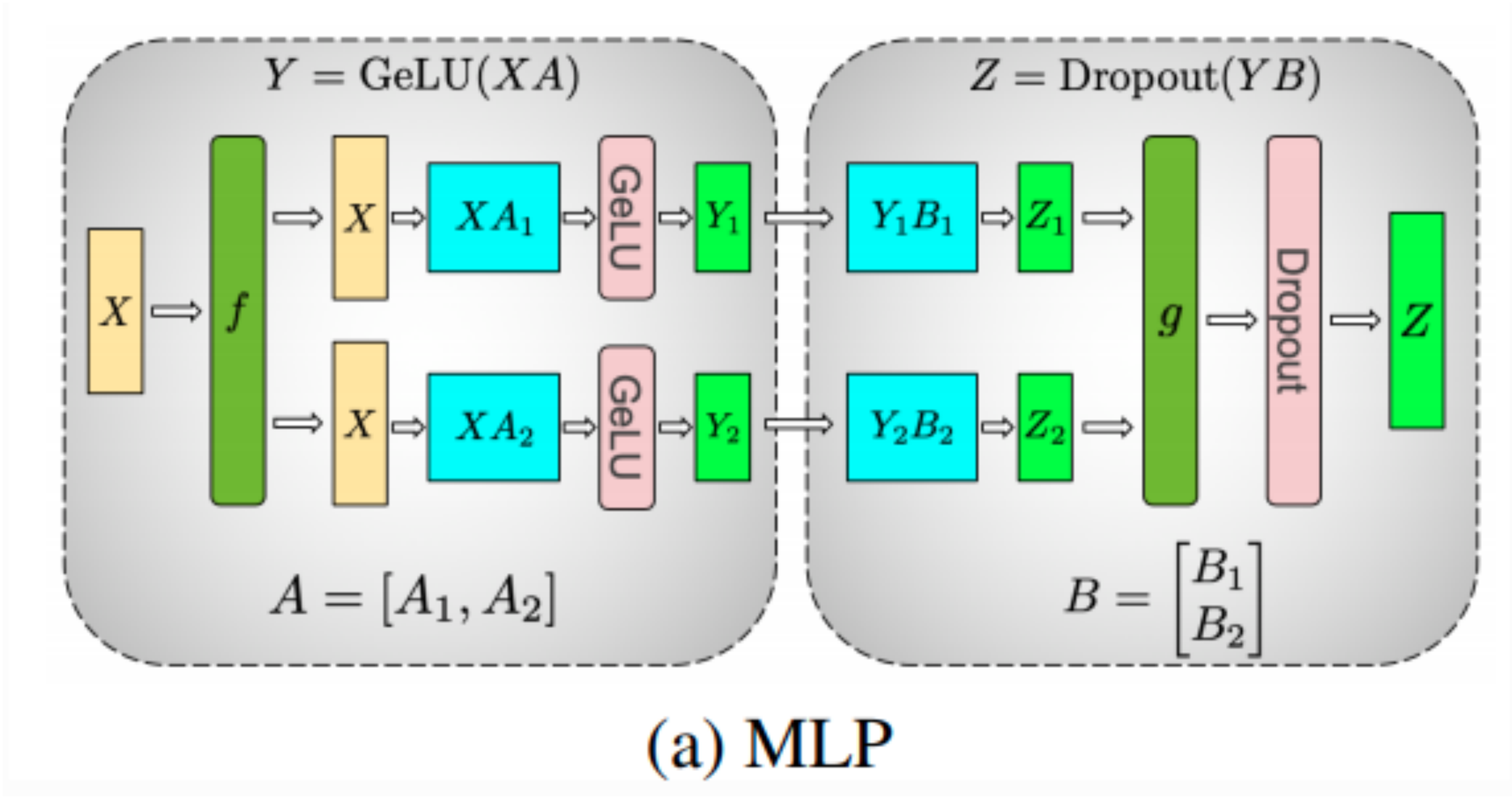


PyTorch: Fully Sharded Data Parallel (FSDP)

Distributed Training: Tensor Parallel



Distributed Training: Tensor Parallel



(a) MLP

Shoeybi et al. Megatron-LM: Training Multi-Billion Parameter Language Models Using Model Parallelism

Distributed Training: Expert Parallel for Mixture-of-Experts